MA 396 Lesson 18 - Board Problems

Extrapolation Methods

1. Use the Extrapolation Algorithm to solve the following IVP.

   $y' = -(y + 1)(y + 3)$ with $0 \le t \le 3$ and $y(0) = -2$. Begin with $h = 0.5$.

Step #1: Use Euler's Method to find w_1. Let $h_0 = \frac{h}{2}$.

Step #2: Find w_2 using the Midpoint Method.

Step #3: Perform Endpoint Correction

$$y_{1,1} = \frac{1}{2}[w_2 + w_1 + h_0 f(a + 2h_0, w_2)]$$

Step #4: Repeat steps 1, 2, and 3 to get $y_{2,1}$ and then average to get $y_{2,2}$. Build the table.

Compare each $y_{i,i}$ to $y_{actual} = -1.53788284$. What is the error? Is it within a TOL=10^{-4}?

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```
In[1]:= Eu[y_, t_, h_] = y + h * f[t, y]
Out[1]= y + h f[t, y]

In[2]:= MP[y0_, y1_, t1_, h_] = y0 + 2 * h * f[t1, y1]
Out[2]= y0 + 2 h f[t1, y1]
```

■ Define your Differential Equation

```
In[3]:= f[t_, y_] = -(y + 1) (y + 3)
Out[3]= (-1 - y) (3 + y)
```

■ Approximate at h/2 using Eulers

```
In[4]:= h0 = .5 / 2
Out[4]= 0.25

In[5]:= w1 = Eu[-2, 0, h0]
Out[5]= -1.75
```

■ Approximate at next step using Midpoint Method

```
In[6]:= w2 = MP[-2, w1, h0, h0]
Out[6]= -1.53125
```

■ End Point Correction

```
In[7]:= y11 = (1 / 2) * (w2 + w1 + h0 * f[.5, w2])
Out[7]= -1.54309082
```

■ Now let's find y21

```
In[8]:= h1 = .5 / 4
        0.125

In[9]:= y0 = -2
Out[9]= -2
```

```
In[10]:= w1 = Eu[y0, 0, h1]
Out[10]= -1.875

In[11]:= w2 = MP[y0, w1, h1, h1]
Out[11]= -1.75390625

In[12]:= w3 = MP[w1, w2, h1 * 2, h1]
Out[12]= -1.640140533

In[13]:= w4 = MP[w2, w3, h1 * 3, h1]
Out[13]= -1.536280959
```

End Point Correction

```
In[15]:= y21 = (1 / 2) * (w4 + w3 + h1 * f[4 * h1, w4])
Out[15]= -1.539150455
```

■ Now let's find y22

```
In[16]:= y22 = y21 + (h1^2 / (h0^2 - h1^2)) * (y21 - y11)
Out[16]= -1.537837001
```

Error

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In[17]:= E1 = Abs[y11 + 1.53788284]
Out[17]= 0.005207980313

In[18]:= E2 = Abs[y22 + 1.53788284]
Out[18]= 0.00004583943947

In[20]:= sol = DSolve[{y'[t] == -(y[t] + 1) * (y[t] + 3), y[0] == -2}, y, t]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. More...

$$\text{Out[20]= } \left\{\left\{y \to \text{Function}\left[\{t\}, -\frac{3 + e^{2t}}{1 + e^{2t}}\right]\right\}\right\}$$

```
In[21]:= y[.5] /. sol
Out[21]= {-1.537882843}
```