MA 396
LSN 19 BOARD PROBLEMS

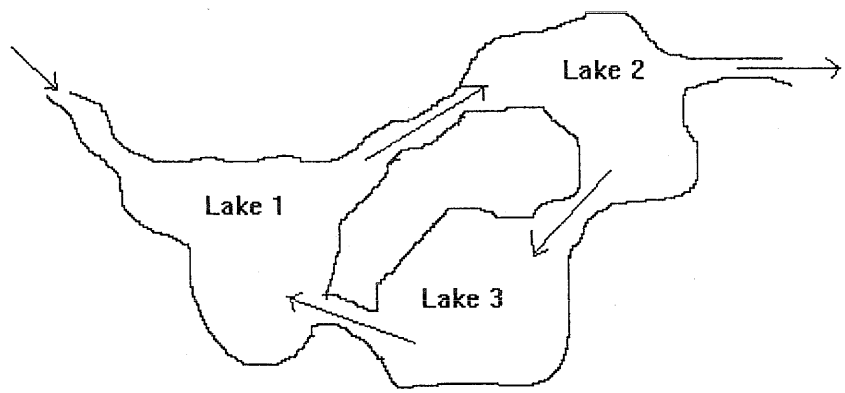
1. Solve the following initial value problems:

   a. $12\,y'' + 5\,y' - 2\,y = 0,\; y(0) = 1,\; y'(0) = -1$

   b. $y'' + 12\,y = 0,\; y(0) = 0,\; y'(0) = 1$

2. Solve the following system of differential equations:

   a. $$x' = 3x - 2y$$ $$y' = 2x - 2y$$



3. Given the system of lakes above and the flow rates below, model the rate of change of pollution with a system of differential equations. What will be the long term behavior of the pollution in the lakes? Why?

Lake 1: 500 gal/min of unpolluted water flowing into the lake from a local river.
700 gal/min is flowing into Lake 2
Lake 1 volume is 100,000 gallons

Lake 2: 500 gal/min is flowing out of the lake into a local river
200 gal/min is flowing into Lake 3
Lake 2 volume is 150,000 gallons

Lake 3: 200 gal/min is flowing into Lake 1
Lake 3 volume is 200,000 gallons

Initial conditions: Lake 1 (0) = 500 pounds of pollution
Lake 2 (0) = 0 pounds of pollution
Lake 3 (0) = 0 pounds of pollution

Lesson 4 Board Problems

1. a) $12y'' + 5y' - 2y = 0$ $\quad y(0) = 1 \quad y'(0) = -1$

CE. $12r^2 + 5r - 2 = 0$

$(4r-1)(3r+2) = 0 \quad r_1 = \frac{1}{4} \quad r_2 = -\frac{2}{3}$

$y = c_1 e^{\frac{1}{4}t} + c_2 e^{-\frac{2}{3}t} \quad y_1 = \frac{1}{4} c_1 e^{\frac{1}{4}} - \frac{2}{3} c_2 e^{-\frac{2}{3}t}$

$1 = c_1 + c_2$

$-1 = \frac{1}{4}c_1 - \frac{2}{3}c_2 \quad c_1 = -.3636 \quad c_2 = 1.3636$

$\left(-\frac{4}{11}\right) \quad \left(\frac{15}{11}\right)$

$$\underline{\underline{y = -\frac{4}{11} e^{\frac{1}{4}t} + \frac{15}{11} e^{-\frac{2}{3}t}}}$$ ans

b) $y'' + 12y = 0$ $\quad y(0) = 0 \quad y'(0) = 1$

CE: $r^2 + 12 = 0 \quad r_1 = 2\sqrt{3}\,i \quad r_2 = -2\sqrt{3}\,i$

$y = c_1 \sin 2\sqrt{3}t + c_2 \cos 2\sqrt{3}t$

$0 = c_2 \quad y' = 2\sqrt{3}\, c_1 \cos 2\sqrt{3}t$

$1 = 2\sqrt{3}\, c_1 \quad c_1 = \frac{\sqrt{3}}{6}$

$$\therefore \underline{\underline{y = \frac{\sqrt{3}}{6} \sin 2\sqrt{3}t}}$$ ans

2. $\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \quad r_1 = 2 \quad r_2 = -1 \quad V_1 = \begin{bmatrix} 1 \\ .5 \end{bmatrix} \quad V_2 = \begin{bmatrix} .5 \\ 1 \end{bmatrix}$

$$\underline{\underline{\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1 e^{2t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}}}$$ ans

3. P_i = pollution in Lake i

$$P_1' = \frac{P_3}{200{,}000 \text{ gal}}\left(200 \text{ gal/min}\right) - \frac{P_1}{100{,}000 \text{ gal}}\left(700 \text{ gal/min}\right)$$

$$P_2' = \frac{P_1}{100{,}000 \text{ gal}}\left(700 \text{ gal/min}\right) - \frac{P_2}{150{,}000 \text{ gal}}\left(500 \text{ gal/min}\right) - \frac{P_2}{150{,}000 \text{ gal}}\left(200 \text{ gal/min}\right)$$

$$P_3' = \frac{P_2}{150{,}000 \text{ gal}}\left(200 \text{ gal/min}\right) - \frac{P_3}{200{,}000 \text{ gal}}\left(200 \text{ gal/min}\right)$$

OR:

$$P_1' = -.007\,P_1 + .001\,P_3$$
$$P_2' = .007\,P_1 - .0047\,P_2$$
$$P_3' = .0013\,P_2 - .001\,P_3$$

$$\begin{bmatrix} P_1' \\ P_2' \\ P_3' \end{bmatrix} = \begin{bmatrix} -.007 & 0 & .001 \\ .007 & -.0047 & 0 \\ 0 & .0013 & -.001 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$r_1 = -.0060 + .00069i \quad r_2 = -.0060 - .00069i \quad r_3 = -.00065$

Notice <u>all</u> eigenvalues have a negative real part.

Therefore, there will be exponential decay. The pollution level will go to zero. Makes sense, since no new pollution is being introduced.