MA 396 Lesson 20 - Board Problems

Higher-Order Equations and Systems of Differential Equations

Perform two iterations of the Runge-Kutte Method for Systems given the second-order differential equation:

$y''-4y'+4y=e^t$ with $0 \le t \le 1$, $h=0.5$, and $y(0)=2$, $y'(0)=4$

First, convert the equation to a system of first-order equations. Then, show that a unique solution exists. Third, approximate the solution, $y(1)$. Finally, solve the second order equation analytically and compare your approximation to the actual solution.

$f_1 = \dot{u}_1 = u_2$ $\qquad u_1(0) = 2$

$f_2 = \dot{u}_2 = e^t - 4u_1 + 4u_2$ $\qquad u_2(0) = 4$

$\frac{\partial f_1}{\partial u_1} = 0 \quad \frac{\partial f_1}{\partial u_2} = 1 \quad \frac{\partial f_2}{\partial u_1} = -4 \quad \frac{\partial f_2}{\partial u_2} = 4$

$\max\left|\frac{\partial f_i}{\partial u_j}\right| = 4 = L \quad \Rightarrow$ solution is unique.

For unique solution, see Maple output.

$y(1) = u_1(1) = 17.284804$ not bad...

Analytically: Characteristic Eqn: $r^2 - 4r + 4 = 0$

$r = 2, 2$

$\therefore \; y_c = c_1e^{2t} + c_2te^{2t}$

$y_p = Ae^t \quad \therefore \; y_p' = y_p''$

$Ae^t - 4Ae^t + 4Ae^t = e^t \quad \Rightarrow A = 1$

$y(t) = c_1e^{2t} + c_2te^{2t} + e^t \qquad y(0) = 2 = c_1 + 1 \qquad c_1 = 1$

$y'(t) = 2c_1e^{2t} + c_2(2te^{2t} + e^{2t}) + e^t \qquad y'(0) = 4 = 2 + c_2 + 1 \qquad c_2 = 1$

$\therefore \; y(t) = e^{2t} + te^{2t} + e^t$

$y(1) = e^2 + e^2 + e = 2e^2 + e \approx \underline{\underline{17.496394}}$ RMS

Board Problem - Lesson 20
RUNGE-KUTTA METHOD FOR SYSTEMS OF DIFFERENTIAL EQUATIONS

t

W 1

W 2

0

2.00000000

4.00000000

$\frac{1}{2}$

5.691753177

12.44558023

1

17.28480433

39.19712253

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