Board Problem – Lesson 26
Linear Shoot Method

The boundary-value problem

$$y'' = 4(y - x), \qquad 0 \le x \le 1, \qquad y(0) = 0 \qquad y(1) = 2$$

has a solution $y(x) = \dfrac{e^2(e^{2x} - e^{-2x})}{e^4 - 1} + x.$

a. Is this solution unique?

$y'' = 4y - 4x \qquad p(x) = 0, \; q(x) = 4, \; r(x) = -4x$

① ALL ARE CONTINUOUS ON $[0,1]$

② $q(x) > 0$ ON $[0,1]$

$\therefore$ THE SOLUTION IS UNIQUE.

b. Letting $h = \dfrac{1}{4}$, solve for $y(x)$. Use algorithm 5.7 to solve the second order IVPs.

$y'' = 4y - 4x \qquad$ LET $\quad y_1 = y$, $\; y_2 = y'$

SO, $\quad y_1' = y_2 \qquad 0 \le x \le 1$

$y_2' = 4y_1 - 4x \qquad y_1(0) = 0$

$y_1'(0) = 0$

$\Rightarrow$

$u(0) =$

$u(.25) =$

$u(.5) =$

$u(.75) =$

$u(1.0) =$

$y_1' = y_2 \qquad 0 \le x \le 1$

$y_2' = 4y_1 \qquad y_1(0) = 0$

$y_1(0) = 1$

$\Rightarrow$

$v(0) = 0$

$v(.25) = .26042$

$v(.50) = .58729$

$v(.75) = 1.06400$

$v(1.0) = 1.81211$

SEE EXCEL SPREADSHEET

LSN 26 Board Problem

t	u	v	Approx	Exact
0	1	0	1	0
0.25	-0.01042	0.26042	0.393714	0.393677
0.5	-0.08729	0.58729	0.824086	0.824027
0.75	-0.314	1.064	1.337163	1.337086
1	-0.81211	1.81211	2	2

Lesson 26 Boards

```
a = 0; b = 1; α = 0; β = 2;
```

```
sol1 = DSolve[{u1'[x] == u2[x], u2'[x] == 4 * u1[x] - 4 * x, u1[a] == α, u2[1] == 0}, {u1, u2}, x]
```

$$\left\{\left\{u1 \to \text{Function}\left[\{x\}, -\frac{e^{-2x}\left(-e^{2}+e^{2+4x}-2e^{2x}x-2e^{4+2x}x\right)}{2\left(1+e^{4}\right)}\right],\ u2 \to \text{Function}\left[\{x\}, -\frac{e^{-2x}\left(e^{2}-e^{2x}-e^{4+2x}+e^{2+4x}\right)}{1+e^{4}}\right]\right\}\right\}$$

```
sol2 = DSolve[{v1'[x] == v2[x], v2'[x] == 4 * v1[x], v1[a] == 0, v2[a] == 1}, {v1, v2}, x]
```

$$\left\{\left\{v1 \to \text{Function}\left[\{x\}, \frac{1}{4}e^{-2x}\left(-1+e^{4x}\right)\right],\ v2 \to \text{Function}\left[\{x\}, \frac{1}{2}e^{-2x}\left(1+e^{4x}\right)\right]\right\}\right\}$$

```
y[x_] = (u1[x] /. sol1) + ((β - u1[b] /. sol1) / (v1[b] /. sol2)) * v1[x] /. sol2
```

$$\left\{\left\{\frac{e^{2-2x}\left(-1+e^{4x}\right)\left(2+\frac{-3e^{2}-e^{6}}{2e^{2}\left(1+e^{4}\right)}\right)}{-1+e^{4}} - \frac{e^{-2x}\left(-e^{2}+e^{2+4x}-2e^{2x}x-2e^{4+2x}x\right)}{2\left(1+e^{4}\right)}\right\}\right\}$$

```
x = a; n = 5; h = .25;
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```
Do[ans = y[x]; Print["At x = ", x, "    y= ", ans]; x = x + h, {i, 1, n}]
```

```
At x = 0    y= {{0}}

At x = 0.25    y= {{0.3936766919}}

At x = 0.5    y= {{0.8240271368}}

At x = 0.75    y= {{1.337086134}}

At x = 1.    y= {{2.}}
```