Board Problem – Lesson 27

Use the Nonlinear Shooting Method to solve the boundary-value problem

$$y'' = y^3 - yy', \qquad 1 \le x \le 2, \qquad y(1) = \frac{1}{2} \qquad y(2) = \frac{1}{3}$$

Compare your results to the exact solution $y(x) = \dfrac{1}{1+x}$.

a) First, write the problem as a system of four first-order equations, including initial conditions.

$y' = v$ $\quad y(1) = \frac{1}{2}$ $\quad z'' = (3y^2 - y')z - yz'$

$v' = y^3 - yv$ $\quad v(1) = -1/6$

$z' = u$ $\quad z(1) = 0$

$u' = (3y^2 - v)z - yu$ $\quad u(1) = 1$

$t_k = \dfrac{\beta - \alpha}{b - a} = \dfrac{1/3 - 1/2}{1} = -\dfrac{1}{6}$

b) Perform one iteration of the Nonlinear Shooting Method. Use the *Mathematica* command NDSolve to solve the system of IVPs. What is the error after one iteration?

Error @ $y[2]$ = .07489

c) Compute a new t_k and perform one more iteration of the Nonlinear Shooting Method.

$t_k = -.25035$

Error @ $y[2]$ =

.0003176

d) What is the error when x = 1.2? x = 1.5? x = 1.8?

See Mathematica

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In[45]:= tk1 = -1 / 6
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$$\text{Out[45]}= -\frac{1}{6}$$

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In[46]:= sol = NDSolve[{y'[x] == v[x], v'[x] == (y[x])^3 - y[x] * v[x],
            z'[x] == u[x], u'[x] == (3 * (y[x])^2 - v[x]) * z[x] - y[x] * u[x],
            y[1] == 1 / 2, v[1] == tk1, z[1] == 0, u[1] == 1}, {y, v, z, u}, {x, 1, 2}]

Out[46]= {{y → InterpolatingFunction[{{1., 2.}}, <>], v → InterpolatingFunction[{{1., 2.}}, <>],
            z → InterpolatingFunction[{{1., 2.}}, <>], u → InterpolatingFunction[{{1., 2.}}, <>]}}

In[47]:= y[1] /. sol

Out[47]= {0.5}

In[48]:= v[1] /. sol

Out[48]= {-0.1666666667}

In[49]:= z[1] /. sol

Out[49]= {-3.130051438 × 10^-21}

In[50]:= u[1] /. sol

Out[50]= {1.}

In[51]:= y[2] /. sol

Out[51]= {0.4082279876}

In[52]:= Error = Abs[(y[2] /. sol) - 1 / 3]

Out[52]= {0.07489465427}

In[53]:= tk2 = tk1 - ((y[2] /. sol) - 1 / 3) / (z[2] /. sol)

Out[53]= {-0.2503496287}

In[54]:= tk2 = -0.2503496287051678

Out[54]= -0.2503496287

In[55]:= sol = NDSolve[{y'[x] == v[x], v'[x] == (y[x])^3 - y[x] * v[x],
            z'[x] == u[x], u'[x] == (3 * (y[x])^2 - v[x]) * z[x] - y[x] * u[x],
            y[1] == 1 / 2, v[1] == tk2, z[1] == 0, u[1] == 1}, {y, v, z, u}, {x, 1, 2}]

Out[55]= {{y → InterpolatingFunction[{{1., 2.}}, <>], v → InterpolatingFunction[{{1., 2.}}, <>],
            z → InterpolatingFunction[{{1., 2.}}, <>], u → InterpolatingFunction[{{1., 2.}}, <>]}}

In[56]:= y[2] /. sol

Out[56]= {0.333017689}
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In[58]:= Error = Abs[(y[2] /. sol) - 1 / 3]
Out[58]= {0.0003156443627}

In[60]:= f[x_] = 1 / (1 + x)
Out[60]= 1/(1 + x)

In[61]:= Error1 = Abs[(y[1.2] /. sol) - f[1.2]]
Out[61]= {0.00006704856513}

In[62]:= Error2 = Abs[(y[1.5] /. sol) - f[1.5]]
Out[62]= {0.0001612699318}

In[64]:= Error3 = Abs[(y[1.8] /. sol) - f[1.8]]
Out[64]= {0.0002534496801}
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