MA 396 – Lesson 2 Board Problems

1. Find the largest interval in which p^* must lie to approximate p with relative error at most 10^{-4} if p is $\sqrt[3]{7}$. $= 1.912931$

$$\left| \frac{1.912931 - p^*}{1.912931} \right| \leq .0001$$

$$-.0001(1.912931) \leq 1.912931 - p^* \leq .0001(1.912931)$$

$$1.912739\overline{7069} \leq p^* \leq 1.913122\overline{8931}$$

2. Find the 3rd Taylor Polynomial for $ln(x)$ about $x_0 = 1$. Find the error bound if $x = 1.1$.

$$f'(x) = \frac{1}{x} \qquad f''(x) = -\frac{1}{x^2} \qquad f'''(x) = \frac{2}{x^3}$$

$$P_3(x) = \ln(x_0) + \frac{1}{x_0}(x-1) - \frac{1}{2!}\left(\frac{1}{x_0^2}\right)(x-x_0)^2 + \frac{1}{3!}\frac{2}{x_0^3}(x-x_0)^3$$

$$= 0 + (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$$

$$R_4(x) = \frac{1}{4!}\left(-\frac{6}{\xi^4}\right)(x-1)^4 \qquad \max @ \ \xi = 1 \qquad \max \xi = \frac{1}{4!}(-6)(1.1-1)^4$$

$$= -.000025$$

$$1 \leq \xi \leq 1.1$$

3. Find the rates of convergence of the following sequences as $h \to 0$:

a) $\lim_{h\to 0} \dfrac{\sin h - h\cos h}{h} = 0$

$O(h^2)$

b) $\lim_{h\to 0} \dfrac{1-e^h}{h} = -1$

$O(h)$

c) $\lim_{h\to 0} \dfrac{\sin h}{h} = 1$

$O(h^2)$

d) $\lim_{h\to 0} \dfrac{1-\cos h}{h} = 0$

$O(h)$

see mathematica work sheet.

■ Problem3a.

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In[4]:= (Series[Sin[h], {h, 0, 4}] - h Series[Cos[h], {h, 0, 4}]) / h
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$$\text{Out[4]}= \frac{h^2}{3} + O[h]^4$$

■ Problem 3b

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In[14]:= (1 - Series[Exp[h], {h, 0, 4}]) / h
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$$\text{Out[14]}= -1 - \frac{h}{2} - \frac{h^2}{6} - \frac{h^3}{24} + O[h]^4$$

■ Problem 3c

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In[15]:= Series[Sin[h], {h, 0, 4}] / h
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$$\text{Out[15]}= 1 - \frac{h^2}{6} + O[h]^4$$

■ Problem 3d

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In[16]:= (1 - Series[Cos[h], {h, 0, 4}]) / h
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$$\text{Out[16]}= \frac{h}{2} - \frac{h^3}{24} + O[h]^4$$