Board Problem – Lesson 27

Consider the boundary-value problem

$$y'' = -3y' + 2y + 2x + 3, \quad 0 \le x \le 1, \quad y(0) = 2, \quad y(1) = 1, \quad h = 0.25.$$

a) Verify that the solution to the tridiagonal linear system is unique.

$$\left.\begin{array}{l} p(x) = -3 \\ q(x) = 2 \geq 0 \\ r(x) = 2x+3 \end{array}\right\} \text{all are continuous on } [0,1]$$

$$L = \max_{0 \le x \le 1} |-3| = 3$$

$$h = .25 < \frac{2}{L} = \frac{2}{3}$$

$\therefore$ unique ans

b) Use the Linear Finite-Difference Algorithm to approximate the solution. Set up the tridiagonal coefficient matrix and the right-hand-side vector.

$$A = \begin{bmatrix} 2 + h^2 q(x_1) & -1 + \frac{h}{2} p(x_1) & 0 \\ -1 - \frac{h}{2} p(x_2) & 2 + h^2 q(x_2) & -1 + \frac{h}{2} p(x_2) \\ 0 & -1 - \frac{h}{2} p(x_3) & 2 + h^2 q(x_3) \end{bmatrix}$$

$x_1 = .25$

$x_2 = .50$

$x_3 = .75$

$$A = \begin{bmatrix} 2.125 & -1.375 & 0 \\ -.625 & 2.125 & -1.375 \\ 0 & -.625 & 2.125 \end{bmatrix}$$

$w_0 = 2$

$w_{N+1} = 1$

$$\vec{b} = \begin{bmatrix} -h^2 r(x_1) + \left(1 + \frac{h}{2} p(x_1)\right) w_0 \\ -h^2 r(x_2) \\ -h^2 r(x_3) + \left(1 - \frac{h}{2} p(x_3)\right) w_{N+1} \end{bmatrix} = \begin{bmatrix} 1.0312 \\ -0.25 \\ 1.0938 \end{bmatrix}$$

$$A\vec{x} = \vec{b}$$

$$\vec{x} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} .8594 \\ .5782 \\ .6848 \end{bmatrix}$$

ans

▪ Board Problem

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In[80]:= p[x_] = -3; q[x_] = 2; r[x_] = 2 * x + 3;

In[81]:= w0 = 2; w4 = 1; h = 1 / 4; x0 = 0;

In[85]:= A = N[{{2 + h^2 * q[x0 + h], -1 + h / 2 * p[x0 + h], 0},
           {-1 - (h / 2) * p[x0 + 2 * h], 2 + h^2 * q[x0 + 2 * h], -1 + (h / 2) * p[x0 + 2 * h]},
           {0, -1 - (h / 2) * p[x0 + 3 * h], 2 + h^2 * q[x0 + 3 * h]}}]

Out[85]= {{2.125, -1.375, 0.}, {-0.625, 2.125, -1.375}, {0., -0.625, 2.125}}

In[86]:= b = N[{{-h^2 * r[x0 + h] + (1 + h / 2 * p[x0 + h]) * w0},
           {-h^2 * r[x0 + 2 * h]}, {-h^2 * r[x0 + 3 * h] + (1 - h / 2 * p[x0 + 3 * h]) * w4}}]

Out[86]= {{1.03125}, {-0.25}, {1.09375}}

In[87]:= Inverse[A] . b

Out[87]= {{0.8594314821}, {0.5782122905}, {0.6847683207}}
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