

## Section 12.3, problem 1 - Hyperbolic Partial Differential Equations

- First we need to find an approximation for the first step...

$$\begin{aligned} w1[xm1_, x0_, x1_] &= (1 - L^2) * f[x0] + (L^2) / 2 * f[x1] + (L^2) / 2 * f[xm1] + k * g[x0] \\ (1 - L^2) f[x0] + \frac{1}{2} L^2 f[x1] + \frac{1}{2} L^2 f[xm1] + k g[x0] \end{aligned}$$

Define your problem

$$\begin{aligned} f[x_] &= \text{Sin}[Pi * x]; g[x_] = 0; h = .25; k = .25; \alpha = 1; L = (\alpha * k) / h; \\ w11 &= w1[0, .25, .50] \\ 0.5 \\ w21 &= w1[.25, .5, .75] \\ 0.7071067812 \\ w31 &= w1[.5, .75, 1] \\ 0.5 \end{aligned}$$

Now we can solve the problem...

$$\begin{aligned} A &= \{\{2 * (1 - L^2), L^2, 0\}, \{L^2, 2 * (1 - L^2), L^2\}, \{0, L^2, 2 * (1 - L^2)\}\} \\ &\quad \{\{0., 1., 0\}, \{1., 0., 1.\}, \{0, 1., 0.\}\} \\ w1 &= \{\{w11\}, \{w21\}, \{w31\}\} \\ &\quad \{\{0.5\}, \{0.7071067812\}, \{0.5\}\} \\ w0 &= \{\{f[.25]\}, \{f[.5]\}, \{f[.75]\}\} \\ &\quad \{\{0.7071067812\}, \{1.\}, \{0.7071067812\}\} \\ w2 &= A.w1 - w0 \\ &\quad \{\{0.\}, \{0.\}, \{-1.110223025 \times 10^{-16}\}\} \\ w3 &= A.w2 - w1 \\ &\quad \{\{-0.5\}, \{-0.7071067812\}, \{-0.5\}\} \\ w4 &= A.w3 - w2 \\ &\quad \{\{-0.7071067812\}, \{-1.\}, \{-0.7071067812\}\} \end{aligned}$$

## Actual Solution and error analysis

```
Act[x_, t_] = Sin[Pi*x]*Cos[Pi*t]  
Cos[\[Pi]t] Sin[\[Pi]x]
```

### ■ Look at the error -

```
Error = Abs[Act[.25, 1] - w4[[1]]]  
{1.110223025 \times 10-16}
```

```
Error = Abs[Act[.5, 1] - w4[[2]]]  
{0.}
```

```
Error = Abs[Act[.75, 1] - w4[[3]]]  
{1.110223025 \times 10-16}
```