MA 396 – Lesson 6 Board Problems

1. a. Use the most appropriate formula to determine approximations that will complete the following table:

x	$f(x)$	$f'(x)$	Eqn used
-3.0	9.367879	-5.87736	4.7
-2.8	8.233241	-5.46893	4.7
-2.6	7.180350	-5.05988	4.6
-2.4	6.209329	-4.65022	4.6
-2.2	5.320305	-4.23991	4.7 (-h)
-2.0	4.513417	-3.82885	4.7 (-h)

SEE ATTACHED MATHEMATICA FILE

b. The data in the exercise above was taken from the function $f(x) = e^{x/3} + x^2$. Find both the actual error and the error bound for x = -3.0 and x = -2.6. Does this seem right? How can you improve on the actual error?

	ERROR BOUND	ACTUAL ERROR	
$X = -3$	6.325×10^{-7}	1.6×10^{-5}	(?)
$X = -2.6$	1.05417×10^{-7}	9.61503×10^{-7}	(?)

ROUND OFF ERROR IN OUR FUNCTION VALUES ABOVE ARE CREATING SIGNIFICANT ERROR.

SEE ATTACHED MATHEMATICA FILE

Lesson 6 Board Problems

Problem 1 - 5 pt formulas

```
In[1]:= fd5[h_, f0_, f1_, f2_, f3_, f4_] = (1 / (12 h)) (-25 f0 + 48 f1 - 36 f2 + 16 f3 - 3 f4)
```

Out[1]= $\frac{-25\,f0 + 48\,f1 - 36\,f2 + 16\,f3 - 3\,f4}{12\,h}$

```
In[2]:= cd5[h_, fm2_, fm1_, f1_, f2_] = (1 / (12 h)) (fm2 - 8 fm1 + 8 f1 - f2)
```

Out[2]= $\frac{8\,f1 - f2 - 8\,fm1 + fm2}{12\,h}$

f'(-3.0)

```
In[7]:= fd5[.2, 9.367879, 8.233241, 7.180350, 6.209329, 5.320305]
```

Out[7]= -5.87736

f'(-2.8)

```
cd5[.2, 8.233241, 7.180350, 6.209329, 5.320305, 4.513417]
```

Out[4]= -5.46893

f'(-2.6)

```
In[5]:= cd5[.2, 9.367879, 8.233241, 6.209329, 5.320305]
```

Out[5]= -5.05988

f'(-2.4)

```
In[8]:= cd5[.2, 8.233241, 7.180350, 5.320305, 4.513417]
```

Out[8]= -4.65022

f'(-2.2)

```
In[9]:= fd5[-.2, 5.320305, 6.209329, 7.180350, 8.233241, 9.367879]
```

Out[9]= -4.23991

■ f'(-2.0)

In[10]:= `fd5[-.2, 4.513417, 5.320305, 6.209329, 7.180350, 8.233241]`

Out[10]= -3.82885

■ Problem 1b

From equation 4.7

In[34]:= `f[x_] = Exp[x / 3] + x^2`

Out[34]= $e^{x/3} + x^2$

In[17]:= `d5[x_] = D[f[x], {x, 5}]`

Out[17]= $\frac{e^{x/3}}{243}$

In[19]:= `N[d5[-3]]`

Out[19]= 0.00151391

In[20]:= `N[d5[-2.2]]`

Out[20]= 0.00197657

Error bound

In[21]:= `(.2^4) / 5 * N[d5[-2.2]]`

Out[21]= 6.32501×10^{-7}

Actual Error

In[28]:= `df[x_] = D[f[x], x]`

Out[28]= $\frac{e^{x/3}}{3} + 2\,x$

In[29]:= `AbsErr = Abs[df[-3] + 5.877357500000001]`

Out[29]= 0.0000160196

From Equation 4.6

In[30]:= `(.2^4) / 30 * N[d5[-2.2]]`

Out[30]= 1.05417×10^{-7}

In[31]:= `AbsErr = Abs[df[-2.6] + 5.059884166666663]`

Out[31]= 9.61503×10^{-7}

■ This is interesting...Can we make our Abs Error smaller?

```
In[35]:= fd5[.2, f[-3], f[-2.8], f[-2.6], f[-2.4], f[-2.2]]

Out[35]= -5.87737

In[36]:= AbsErr = Abs[df[-3] + 5.877374061374922]

Out[36]= 5.41765 × 10^-7
```