MA 396 – Lesson 8 Board Problems

1. Approximate the following integrals using (i) the Trapezoidal rule and (ii) Simpsc n's rule:

$$\int_2^2 x^3 e^x \, dx$$

2. Let $n = 4$, approximate the same integral using (i) the Composite Trapezoidal rule and (ii) the Composite Simpson's rule.

3. Find the error bound for the composite Simpson's rule solution in problem 2 abov e. What is the reduced error bound if we make n = 8 and n = 16?

In[1]:=
```
T[x_, h_] = (h / 2) * (f[x] + f[x + h])
```

Out[1]= $\frac{1}{2}\, h\, (f[x] + f[h + x])$

In[2]:=
```
S[x_, h_] = (h / 3) * (f[x] + 4 * f[x + h] + f[x + 2 * h])
```

Out[2]= $\frac{1}{3}\, h\, (f[x] + 4\, f[h + x] + f[2\, h + x])$

In[3]:=
```
M[x_, h_] = 2 * h * f[x + h / 2]
```

Out[3]= $2\, h\, f\left[\frac{h}{2} + x\right]$

In[4]:=
```
CT[x_, h_, n_] = (h / 2) * (f[x] + 2 * Sum[f[x + j * h], {j, 1, n - 1}] + f[x + n * h])
```

Out[4]= $\frac{1}{2}\, h \left(f[x] + f[h\, n + x] + 2 \sum_{j=1}^{-1+n} f[h\, j + x]\right)$

In[5]:=
```
CS[x_, h_, n_] =
  (h / 3) * (f[x] + 2 * Sum[f[x + 2 * (j * h)], {j, 1, (n/2) - 1}] + 4 * Sum[f[x + (2 * j - 1) * h], {j, 1, n/2}] + f[x + n * h])
```

Out[5]= $\frac{1}{3}\, h \left(f[x] + f[h\, n + x] + 2 \sum_{j=1}^{-1+\frac{n}{2}} f[2\, h\, j + x] + 4 \sum_{j=1}^{\frac{n}{2}} f[h\, (-1 + 2\, j) + x]\right)$

In[6]:=
```
f[x_] = x^3 * Exp[x]
```

Out[6]= $e^x\, x^3$

In[7]:=
```
T1 = N[T[-2, 4]]
```

Out[7]= 116.06

In[8]:=
```
S1 = N[S[-2, 2]]
```

Out[8]= 38.6865

In[9]:=
```
T2 = N[CT[-2, 1, 4]]
```

Out[9]= 31.3653

In[10]:=
```
S2 = N[CS[-2, 1, 4]]
```

Out[10]= 22.4771

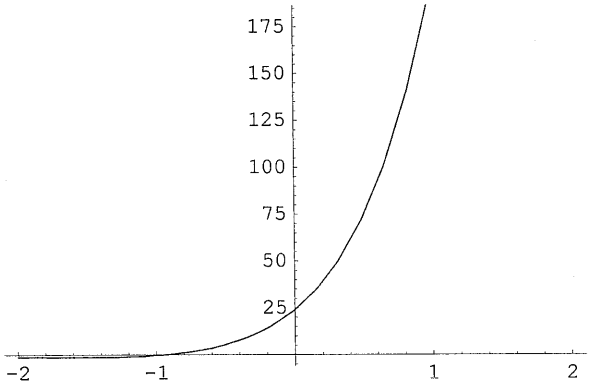
In[11]:=
```
Problem 3
```

Out[11]= 3 Problem

In[12]:=
```
d4[x_] = D[f[x], {x, 4}]
```

Out[12]= $24\, e^x + 36\, e^x\, x + 12\, e^x\, x^2 + e^x\, x^3$

In[13]:= **Plot[d4[x], {x, -2, 2}]**



Out[13]= - Graphics -

In[14]:= **Error bound**

Out[14]= bound Error

In[15]:= **(2 - (-2)) / 180 * 1 * d4[2]**

Out[15]= $\frac{152\ e^2}{45}$

In[16]:= **N[%]**

Out[16]= 24.9586

In[17]:= **What is the error bound if we make n = 8?**

```
General::spell1 :
 Possible spelling error: new symbol name "error" is similar to existing symbol "Error". More...

Syntax::tsntxi : "8?" is incomplete; more input is needed. More...

What is the error bound if we make n = 8?
```

In[17]:= **(2 - (-2)) / 180 * (1 / 2) ^ 4 * d4[2]**

Out[17]= $\frac{19\ e^2}{90}$

In[18]:= **N[%]**

Out[18]= 1.55991

In[19]:= **(2 - (-2)) / 180 * (1 / 4) ^ 4 * d4[2]**

Out[19]= $\frac{19\ e^2}{1440}$

In[20]:= **N[%]**

Out[20]= 0.0974945