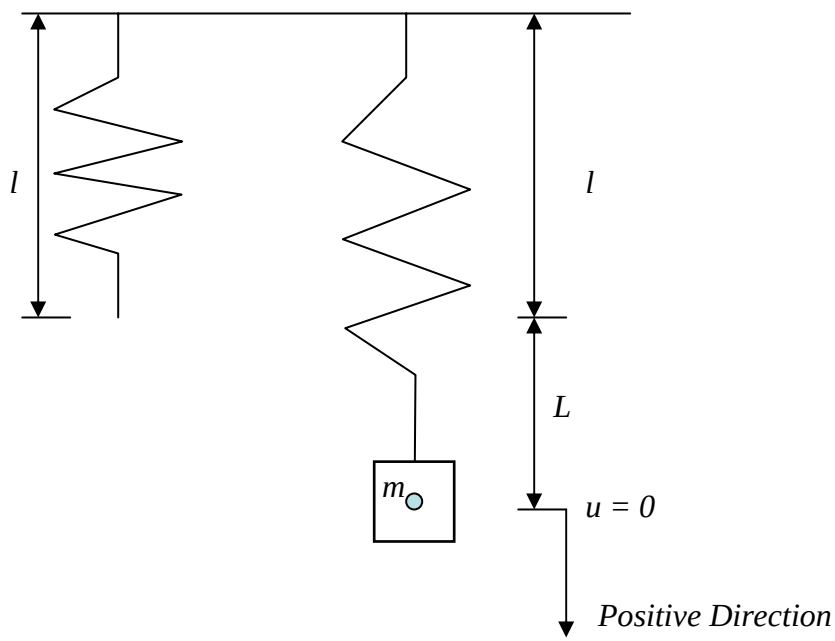
Due: 5 April 2005

MA396

Project 4

Situation – You have a spring of length *l*, called the natural length, and you are going to hook an object with mass *m* up to it. When the object is attached to the spring, the spring will stretch a length of *L*. We will call the equilibrium position the position of the center of gravity for the object as it hangs on the spring with no movement.

Below is sketch of the spring with and without the object attached to it. As denoted in the sketch we are going to assume that all forces, velocities, and displacements in the downward direction will be positive. All forces, velocities, and displacements in the upward direction will be negative.



Also, as shown in the sketch above, we will measure all displacement of the mass from its equilibrium position. Therefore, the $u = 0$ position will correspond to the center of gravity for the mass as it hangs on the spring and is at rest (*i.e.* no movement).

The following forces will be acting on our object -

1. Gravity, F_g

The force due to gravity will always act upon the object of course. This force is $F_g = mg$.

2. Spring, F_s

We are going to assume that Hooke's Law will govern the force that the spring exerts on the object. This force will always be present as well and is

$$F_s = -k(L + u).$$

Hooke's Law tells us that the force exerted by a spring will be the spring constant, $k > 0$, times the displacement of the spring from its natural length.

3. Damping, F_d

Dampers work to counteract any movement and can be modeled as

$$F_d = -\gamma u'$$

where, $\gamma > 0$ is the damping coefficient.

4. External Forces, $F(t)$

We typically call $F(t)$ the forcing function and it can take many forms.

Requirement 1:

Using Newton's Second Law, develop a second order differential equation that models the motion of the object. Assume all four forces discussed above are acting on your object. Rewrite the second order differential equation as a system of first order differential equations.

Requirement 2:

A 16 lb object stretches a spring 8/9 ft by itself. There is no damping and no external forces acting on the system. The spring is initially displaced 6 inches upwards from its equilibrium position and given an initial velocity of 2 ft/sec downward. By hand, find the displacement at any time t, $u(t)$. Confirm your answer using *Mathematica*. What is the position of the object at $t = 1.5$ sec? Plot your results. Describe the behavior of your spring-mass system.

Requirement 3:

Take the spring and mass system from requirement 2 and attach a damper to it that will exert a force of 5 lbs when the velocity is 2 ft/s. Using the method for find numerical solutions to a system of differential equations discussed in Section 5.9 of the text, estimate the position of the object at $t = 0.3$. However, rather than using Runge-Kutta as the base algorithm, use the Midpoint Method. Let $h = 0.1$. Compare your numerical result with the "exact" solution found using *Mathematica*. How can you improve on the accuracy of your estimation?

Requirement 4:

A 3 kg object is attached to a spring and will stretch the spring 400 mm by itself. Additionally, a damper will exert a force of 45 Newtons when the velocity is 50 cm/sec. A forcing function of the form

$$F(t) = 10\sqrt{t}\cos(5t)$$

is attached to the object. The object is initially displaced 20 cm downward from its equilibrium position and given a velocity of 10 cm/sec upward. Attempt to find the solution using *Mathematica*. Are you successful? If not, why is Mathematica having difficulties? Approximate the solution at t = 2.0 using Algorithm 5.7. Let N = 20 and N = 40. Compare these results with the Mathematica command NDSolve.

Format: Submit a typed report that addresses the four requirements. You should include an Executive Summary, a discussion of each requirement, with work or computations done in order to answer each question, and your conclusions. Attach any numerical analysis approximations as tabbed appendices to your report. If you used appendices, refer to them in your analysis.