Lsn 29 Board Problem:

1. Let X have the exponential distribution with parameter 2. Let U have the Uniform(0,1) distribution. Let n = 4.

a. Find the distribution of the maximum order statistic for X

$n=r=4 \quad f(x) = 2e^{-2x} \quad 0<x<\infty$

$$f_4(x) = \frac{4!}{(4-1)!(4-4)!}(F(x))^3 f(x) (S(x))^0 = 4(1-e^{-2x})^3(2e^{-2x})$$

$$= 8e^{-2x}(1-e^{-2x})^3 \quad 0<x<\infty$$

b. Find the distribution of the r = 2 order statistic of U.

$n=4 \quad r=2 \quad f(x)=1 \quad F(x)=x \quad S(x)=1-x$

$$f_2(x) = \frac{4!}{2!1!} x\cdot(1-x)^2 = 12x(1-x)^2, \quad 0<x<1$$

2. A manufacturer claims that his batteries will last an average of 48 hours of continual us on a SINGARS radio system. You assume the lifetime is exponentially distributed and place four new batteries in continuous use. The first battery fails after only 10 hours. What do you conclude about the manufacturer's claim?

$f(x) = \frac{1}{48}e^{\frac{-x}{48}} \quad n=4 \quad r=1$

$$f_1(x) = \frac{4!}{3!}(F(x))^0 f(x) (S(x))^3 = 4\cdot\frac{1}{48}e^{-x/48}\left(e^{\frac{-x}{48}}\right)^3$$

$$= \frac{1}{12}e^{-\frac{1}{12}x}$$

We expect the first battery to follow this distribution, exponential with a mean of 12 hours. The first failing at 10 hours is likely within tolerance, although low.