MA 476 WPR 1 6 March 2003 **NAME:** Solution

1. (30 Points) Consider the random variable X that has CDF

$$F(x) = 1 - (1+x)e^{-x} \quad 0<x<\infty.$$

a. Find the PDF of this RV

$$f(x) = F'(x) = \frac{d}{dx}\left(1-(1+x)e^{-x}\right) = xe^{-x} \qquad 0<x<\infty$$

b. Find the SF of this RV

$$S(x) = 1 - F(x) = 1-\left(1-(1+x)e^{-x}\right)$$
$$= (1+x)e^{-x} \qquad 0<x<\infty$$

c. Find the hazard function of this RV

$$h(x) = \frac{f(x)}{S(x)} = \frac{(x)e^{-x}}{(1+x)e^{-x}} = \frac{x}{1+x} \qquad 0<x<\infty$$

2. (10 Points) Refer to suggested problem 3.42 on page 99 of your text. Find *P(4< Y < 6).*

$$F(y) = 1 - \frac{9}{y^2} \qquad 3<y$$

$$P(4<Y<6) = F(6) - F(4) = \left(1-\frac{9}{6^2}\right) - \left(1-\frac{9}{4^2}\right) = 5/16$$

$$= .3125$$

3. (30 Points). Consider the Weibull random variable with CDF as follows:

$$F(x) = 1 - e^{-(\lambda x)^{\kappa}} \quad \text{for } 0 < x < \infty \text{ and } \lambda, \kappa > 0.$$

Let $\lambda = 0.05$ and $\kappa = 2$. Use Monte Carlo simulation to find the Weibull variates that correspond to the uniform random numbers 0.21 and 0.69.

$u = 1 - e^{-(\lambda x)^k}$

$-u + 1 = e^{-(\lambda x)^k}$

$-\ln(1-u) = +(\lambda x)^k$

$\sqrt[k]{-\ln(1-u)} = \lambda x$

$\frac{1}{\lambda}\sqrt[k]{-\ln(1-u)} = x = F^{-1}(u)$

$F^{-1}(.21) = 9.71025$

$F^{-1}(.69) = 21.6442$

4. (30 Points) Consider the random variable X that has PDF

$$f(x) = xe^{-x} \quad 0<x<\infty.$$

. Find E(X) and V(X).

$E(X) = \int_0^\infty x \cdot xe^{-x}\, dx = \underline{\underline{2}}$

$E(X^2) = \int_0^\infty x^2 \cdot xe^{-x}\, dx = 6$

$V(X) = E(X^2) - \mu^2 = 6 - 2^2 = \underline{\underline{2}}$

5. (25 Points) The moment generating function for the Erlang(2,1) random variable is

$$M(t) = (1-2t)^{-2}.$$

Use this MGF to find the mean of the distribution.

$$\mu = m'(t)\Big|_{t=0} = \frac{d}{dt}(1-2t)^{-2}\Big|_{t=0} = \frac{-4}{(2t-1)^3}\Big|_{t=0}$$

$$= \frac{-4}{-1} = \underline{\underline{4}}_{\text{ANS.}}$$

6. (20 Points) The Rayleigh distribution is outlined in problem 6.20 on page 213 of your text. Assume the information given in (a) and (b) is true (it is...). Use the method of moments to find an estimate for the parameter α. For empirical data, use this set of numbers; [12.3, 9, 15].

$$\mu = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}} \qquad \bar{X} = 12.1$$

Set $\mu = \bar{X}$ solve for α

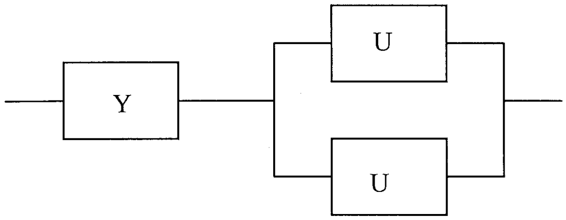
$$12.1 = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

$$2(12.1) = \sqrt{\pi/\alpha}$$

$$[2(12.1)]^2 = \frac{\pi}{\alpha}$$

$$\alpha = \frac{\pi}{[2(12.1)]^2} = .00536$$

7. (30 Points) Consider the following reliability block diagram



Assume U~Uniform(0, 4) and Y~Exponential(1/10).

Find the CDF of the system; be sure to show the support of the new random variable.

$U_{para} = max(u, u)$

$F_{U_p} = F_u \cdot F_u = \frac{x}{4} \cdot \frac{x}{4} = \frac{x^2}{16} \qquad 0 < x < 4$

$S_{System} = S_Y \cdot S_{u_p} = e^{-\frac{x}{10}}\left(\frac{x^2}{16} + 1\right) \qquad 0 < x < 1$

$F_{System} = 1 - S_{System} = 1 + \frac{x^2}{16} e^{-x/10} - e^{\frac{-x}{10}} \qquad 0 < x < 4$

8. (2 extra credit points) Find P(X<0.95) for the system in number 7 above.

$F(.95) = 1 + \frac{(.95)^2}{16} e^{-.95/10} - e^{\frac{-(.95)}{10}} = .1419$