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CUSUM AND HUMAN BEHAVIOR

by

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Can human behavior be quantified to determine a person's commitment in a given society? An approach to providing insight into this question is proposed by applying statistical process control charts. The Cumulative Sum (CUSUM) control chart is a powerful tool that detects when a change may have occurred in a stochastic process. A human's participation within a socially connected group can be measured as a stochastic process. However, unlike manufacturing processes, it is difficult to tie a cost function to changes in human social behavior. This requires the analyst to estimate CUSUM control chart parameters when given a fixed false alarm probability.

Monte Carlo simulations and response surface methods are used to determine a function for the appropriate CUSUM parameters based on a user defines type-I error. These control chart parameters can be applied to the analysis of human behavior to determine when the participation level of an individual changes over time. After the most likely time has been identified, we can investigate factors causing the change in behavior. Patterns can be identified to show what causes individual or group behavior. The ability to estimate control chart parameters has applications in quality engineering, organizational behavior, and social network analysis.

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CUSUM AND HUMAN BEHAVIOR

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I. INTRODUCTION

A man walks down the streets of Iraq on his way home from his job at a local shop. He has four children, a wife, and his parents depending on his sub-par paycheck. Needless to say, he has been struggling to make ends meet in this war torn country, but is happy to have a life where he can work and provide for his family, even if he can barely provide just enough to support his dependents. Nevertheless, he firmly believes that his life will improve and opposes insurgent activity. After all, the insurgency only brings harm to his family. However, on this ill-fated walk home, he is confronted by three unfamiliar men while he is taking a shortcut through an alleyway.

He never returns home that night and his family fears the worst. They fear that he was caught in a fire fight between insurgents and coalition troops or is the victim of an attack. But he is safe, for now. The three men throw a blindfold over his eyes, throw him into a van parked in the alley, and drive away. They drive for about an hour, he does not know exactly how long, but he's scared. The van stops and he's dragged out and tied to a chair in an unfamiliar room. He sits there for a while wondering if he will ever see his family again, wondering how they will survive without his support. He just begins to sob when a large man walks in the door, pulls up a chair, and talks with him for a while. Three days later, he is dropped off at his doorstep with a roll of American cash in his front pocket. He walks in the door and surprises his family. They are overjoyed to see him and ask where he had been. Of course, he cannot tell them, he only says that he had an opportunity to make some money and could not come home. Three weeks later, this man lies dead on the side of the road after being caught trying to emplace a road side bomb along a major highway leading into Baghdad.

Why did this man become part of a terrorist organization when he opposed it only a few weeks before? How did he become part of an organization and involved with people that he never met in only three weeks? Every organization has members who are involved at different levels. As time progresses, members become either more vested or decrease their participation in the organization. The movement of the members from different levels of involvement determines the strength and health of the organization [1]. If there are a disproportional number of members in any of the participation levels, the organization can experience turmoil and inefficiency because either too many or too few leaders exist [1]. The existence of the participation levels and the proportional dispersion of members

is a direct result of the leadership's ability to provide sufficient services and meaning that are tailored to the members' needs and goals [1]. Therefore, organizations need to divert resources to support not only their organizational goals, but to support their members in order to ensure the strength and health of the organization.

A. OBJECTIVE

The primary objective of this study is to present a surface function designed to assist an analyst in determining the appropriate control limit for the Cumulative Sum Control Chart (CUSUM) (See Chapter II, Section B). Given the analyst's acceptance of type-I error, the surface will allow the control limits to be calculated without the use of a loss function. A loss function is a function that allows an analyst to estimate the costs of decisions and events. It allows the control limits of the CUSUM to be determined based on the possibility of costs associated with an shift in process parameters. However, not all processes have consequences associated with changes to process parameters. Therefore, this function will allow the CUSUM to be applied to situations in which loss functions do not exist such as organizational behavior and social network analysis.

B. RESEARCH QUESTIONS

This study will address the following research questions:

1. Can a surface function be modeled to estimate the appropriate control limits based on type-I error probability?
2. Does the control limit evaluated from the surface provide accurate statistical information when the CUSUM technique is applied to a set of data?
3. How can this model advance our understanding of organizational behavior and social networks?

C. PARTICIPATION STRUCTURE

An organization is comprised of individuals who are participating at different levels. There are three basic levels of involvement: peripheral, active, and core-group. Each group becomes smaller and more select as influence and responsibility increase [3]. The periphery is the largest and most diverse group and the core-group is the smallest and most select group in the community (Figure 1). The active group is comprised of individuals who

are more consistently active; however, they do not hold leadership roles. An individual becomes a member of the core-group by making a conscience decision to become either a formal or informal leader within the organization. The existence of the core-group is essential to the success and development of the organization [3].

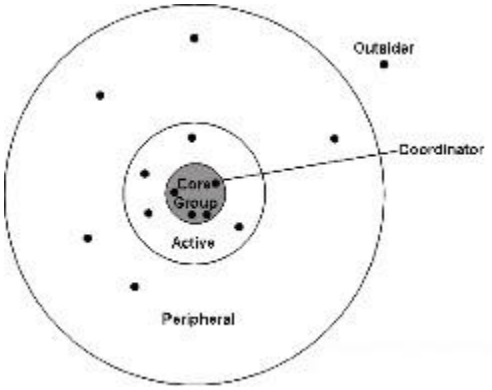

Figure 1. Participation in a Community of Practice [3]

Individuals within these different participation levels move from one level to another by either becoming more or less active in a community of practice. A community of practice is a type of informal social system or organization characterized by voluntary human interactions and relationships [1]. In most organizations, the lines that define these levels are not clear and social groups may not exclusively exist within one participation level. Just because an individual is currently within the peripheral, does not mean he/she cannot interact with members in the active or core group. As a result, the time when that individual becomes more active in the organization is difficult to determine. Furthermore, the reasons for the increase in participation is more difficult, if not impossible, to determine if they time of movement is unknown. This presents a problem for the people interested in determining how to better serve their members or understand what people value in their organization.

D. COMPANYCOMMAND AND THE ARMY

Modeling human behavior and other qualitative processes can be directly related to the study of communities of practice. The development of a surface function to determine appropriate control limits for a process that does not necessarily have a loss function can be very valuable. One of these communities of practice, which the CUSUM technique can possibly applied to, is the Army and CompanyCommand.

Before getting to CompanyCommand, a brief description of Army organization at the company level is necessary. A company (also troop, battery, or detachment) is an organizational unit within the Army. The company commander is the individual who is in command of the company which consists of approximately 100 soldiers. The commander is usually five to eight years experienced in the Army and leads the organization for an average of eighteen months [1]. This is the first level where leaders are granted full command authority. The Global War on Terrorism (GWOT) has changed the operating environment so power and responsibility has been increasingly delegated to the company level. The increased responsibility placed on company commanders has given them more responsibility and power than ever before.

This increase has created a need to develop the Army's leadership at the company level, thus CompanyCommand was created. CompanyCommand is a web-based community, or professional forum, that facilitates peer-peer leadership development in the Army. This community provides an opportunity for past, present, and future company commanders to interact with each other to share professional experiences, leadership advice, and gain insight on dilemmas and challenges. The stated purpose of the CompanyCommand forum, according to Lieutenant Colonel Tony Burgess, is: "To improve the effectiveness of company commanders, and to advance the practice of company command [4]." The connections and discussions that are established on CompanyCommand are not only posted on the website but extend to private peer-peer contact through e-mail and telephone conversations [4]. The relationships and professional development that CompanyCommand facilitates, creates an ever growing relationship between the leaders of the past, present, and future. Members can participate in many different ways from simply logging in to becoming topic leads, command contact, or point-men [1].

E. AL QAEDA

Al Qaeda is the radical Islamic terrorist organization responsible for the horrific attack on the World Trade Center buildings on September 11, 2001. Al Qaeda is a social network with many individuals who participate at different levels. In 2007, Major Ian McCulloh applied CUSUM control charts to analyze the activity of the Al Qaeda network. His study identified a shift in some of the network measures between 2000 and 2001 [5]. Furthermore, he identified that the most likely increase of islamic extremist activity occurred in 1997. When investigated, he learned that there were several events related to Al Qaeda

and Islamic extremism that occurred in 1997 including the massacre of 58 foreign tourists and at least four Egyptians in Luxor, Egypt. Additionally, Al Qaeda leadership, Osama Bin Laden and Ayman al-Zawahiri, publically reunited in 1998. The United States embassies in Tanzania and Kenya were bombed by Al-Qaeda as well [5].

McCulloh explains that his control limit, h, and optimality constant, k, were arbitrarily set at 4 and 0.5 respectively for all of the control charts. He was forced to utilize these common values because there was no method to determine the appropriate values of these parameters [5]. These parameters determine how quickly the chart signals that a change may have occurred. Even though the analysis determined the most likely increase in radical Islamic action occurred in 1997, the year of many significant events, it did not signal that a change may have occurred until 2000 and 2001. Therefore, if the analysis would have been conducted throughout the time period of interest, the signal would have occurred too late to attempt to prevent an attack like the one on the World Trade Center. Had the parameters been adjusted to allow the chart to signal quicker, insight to increased Al Qaeda activity could have been provided before the attacks on September 11th.

F. PROBLEMS WITH QUANTITATIVE MODELING

Many organizations track statistics focused on their member's participation. However, these statistics are merely data without statistical analysis. To date, the analysis of these statistics involving human behavior has taken a qualitative approach [1]. The approach has been implemented by simply looking at raw data to determine which members were more vested in their organizations. This was followed by conducting interviews to determine why certain individuals participated at their level [1].

In order to provide statistical significance to the study of human behavior, the qualitative approaches need to be supplemented with quantitative statistical analysis. This can be achieved by the application of CUSUM control charts to determine the most likely time when the behavior of an individual or group changes. Furthermore, in order for the CUSUM chart to provide statistically significant information, the control limits must be adjusted to the appropriate level of sensitivity based on the process being analyzed.

To date, the determination of appropriate control limits for the CUSUM chart has been limited to processes that involve the potential of loss such as business or financial processes. The control limits can be determined by the application loss functions designed to incorporate the risk of incurring costs to a decision. Appropriate control limits allow

the analyst to adjust the sensitivity of the control chart to account for the consequences of a loss [6]. However, when studying other stochastic processes that do not involve loss potential, such as human behavior, the current methods of incorporating loss functions cannot assist in obtaining the appropriate control limits. Consequently, the determination of appropriate control limits for these processes is highly arbitrary. Because not all processes have an applicable loss function, the application of CUSUM charts does not present strong statistical value.

Adjusting the control limits of CUSUM charts determine how quickly the chart signals that a change may have occurred. The CUSUM is designed to detect small changes in a process. As the control limits change there is a trade-off between the speed that a change is detected and the probability of type-I error [7]. When the CUSUM control chart makes a prediction of the most likely time a process changes with the appropriate control limits, it makes the prediction with statistical significance [8].

Predictions associated with human behavior to determine the most likely times an individual becomes more vested in an organization can be applied to many different situations. These situations include everything from determining how to appeal to members of the organization in order to increase participation to deterring people from becoming more committed to an organization. Today's world is faced with a new enemy, those who conduct terrorism. New people become associated with terrorism and their commitment to their respective terrorist cells can increase in a variety of ways. This process could also present insight and quantitative values to determine when individuals become more vested in their terrorist cell. Once the timeline of most likely significant events has been created, investigations can be conducted to determine the reasons behind the increased participation. Efforts can be directed to deter or counter the increased involvement the terrorists have within the cell. This can provide invaluable information and help our government and military defeat the terrorist threat that our world faces today. Furthermore, as the Al-Qaeda example illustrated, this concept can be applied to the activity of an organization over time.

II. BACKGROUND

A. PREVIOUS RESEARCH: COMMUNITIES OF PRACTICE AND THE CORE GROUP PHENOMENON

Research in this area has been limited to qualitative statistical analysis. Burgess conducted a study that ”explored the process whereby members of the core-group traveled from the periphery of the community to the core [1]” and investigated what their experiences meant to them. Once Burgess identified the members of the core-group within the community of practice, CompanyCommand, he utilized an in-depth interview method that consisted of three, ninety-minute interviews with participants who were within the core-group. He identified the participants in his study by identifying approximately fifty potential core-group members and selecting ten based on the principle of ”sufficiency” to maximize variance within the sample [9][10]. His decision was based on three factors: amount of time in the core-group, basic branch or specialty field, and current status as a company commander. Each interview had a different objective aimed at determining how they became core-group members and what their experiences meant to them [1]. His study successfully determined the significance of the participants' journey from the periphery to the core. He was able to develop a rough timeline outlining the process in which a member becomes a core-group member (Figure 2).

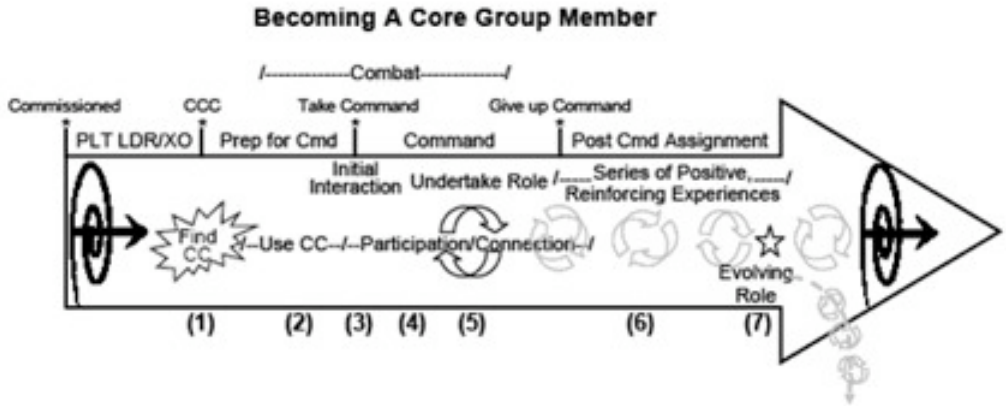

Figure 2. Becoming A Core Group Member [1]

Burgess describes the basic storyline of a participant moving from the periphery to the core through seven steps [1]:

(1) Discovery of CompanyCommand around the time he/she attends the captain's career course

(2) Uses it to prepare for command and then continues to use it throughout command

(3) Receives an initial interaction between the officer and members of the core-group

(4) Pattern of increasing participation and interaction with core-group members

(5) Receives an invitation to take part in a leadership role

(6) A series of meaningful experiences that have a cumulative, positive effect

(7) Evolvement into greater leadership roles within CompanyCommand or a gradual disengagement from CompanyCommand

Although the study achieved its purpose, it did not possess quantitative aspects to determine when those individuals increased their participation within CompanyCommand. The application of quantitative statistical analysis by using statistical process control will allow CompanyCommand to determine the most likely times when an individual increased or decreased his/her participation level. When these times are found, the CompanyCommand staff can investigate the cause of the change in participation by the individual member within the CompanyCommand forum. However, without the incorporation of appropriate parameters, the efficiency and effectiveness of determining when an individual changes his participation habits undermines the results of the test. Furthermore, current methods of the application of a loss function to determine the appropriate control limits of CUSUM control charts cannot be applied to human behavior. Therefore, the estimation and modeling of the parameters must be completed to allow the analyst achieve significant results based on his acceptance of type-I error.

B. STATISTICAL PROCESS CONTROL

Statistical process control (SPC) is a statistical tool used to determine if a change in the mean of a population or data set has changed over time [11]. It has been applied in business to address quality control issues in order to ensure that work flow and manufacturing maintain the proper quality levels [11]. SPC can also be used in financial and economic situations as well. The possibilities of different applications of SPC extend much farther than quality control or financial data sets. Sciences such as social networking present an interesting question: can the activity of an individual or group be modeled using SPC?

SPC utilizes control charts as a graphical representation of the performance of the process over time. Control charts are a medium that can help an analyst determine if the process remains in-control or if it becomes out-of-control. An in-control process is one in which the process mean or distribution does not change. If the mean or distribution of the process changes, it then becomes out-of-control. SPC signals that a change may have occurred when the test statistic falls outside of the control limits. The control limits are set at a specified level of uncertainty depending on the process in question [8]. The ability of an analyst to quickly determine when a change may have occurred in a process allow control charts to be a widely accepted method for distinguishing between normal process variation and unusual variation due to a special cause [12].

1. Definition of Terms

Throughout this paper statistical terms will be used in order to articulate the CUSUM control chart. The common terms associated with the CUSUM include the optimality constant (k), decision interval (h), type-I error (α), and Average Run Length (ARL = $\frac{1}{\alpha}$). The optimality constant is a parameter of the CUSUM test statistic that adjusts the value of the statistic. It allows the analyst to adjust the ARL of the control chart to fit the requirements of the process. The decision interval, also known as the control limit, is the measure to which the test statistic is compared to. When the statistic exceeds the decision interval, the CUSUM signals that a change may have occurred. Similar to the optimality constant, the decision interval can be adjusted to fit the process. The purpose of this study is to find the optimal decision interval based on the process and the analyst's acceptance of type-I error probability.

Type-I error is known as a false positive. When a type-I error arises, the chart signals that a change may have occurred, however, an investigation shows that a change in the process parameter did not actually occur. The acceptance of the probability type-I error depends on many factors including the personality of the analyst, wishes of the client, and type of process being analyzed. The probability of type-I error determines the ARL of the control chart. The ARL is the average amount of runs before an in-control process signals that a change may have occurred, when it was actually type-I error. In order to apply the CUSUM, the analyst must assume that the process is in-control. An in-control process is a process in which the process parameters remain constant and any variation is the result of normal process variation [12].

2. Step and Trend Changes

When a process is considered out-of-control, a change in the process parameter has occurred. There are two types of changes: step-changes and trend-changes. Step changes occur when the process parameter abruptly changes value. This model assumes that the process is initially in-control where the mean $\mu = \mu_0$ and μ_0 is known. When a step-change occurs, $\mu \neq \mu_0$ and the mean suddenly changes to μ_1. At this point, we adjust our process where $\mu = \mu_1$, where μ_1 is the new sample mean [12].

Trend-changes occur when there is a correlation among the data points. Ignoring trends within the control chart can cause an inaccurate estimation of the control limits for the control chart. However, if trends arise during the study, it can suggest that there is a steady change in the process parameter over time [13].

3. False Alarms

A false alarm occurs when the control chart signals that a change may have occurred within an in-control process. The signal must be investigated to determine if a change has occurred. However, because the process is in-control, the parameter did not change. Because control charts are based on statistical distributions and the control limits are developed at a specific level of certainty, it is possible for a point to fall outside of the control limits and still follow the original distribution centered on μ_0. The utilization of a control chart that produces too many false alarms makes it inefficient because each signal must be investigated to determine whether or not the process is still in-control

If the process is determined to be out-of-control after an investigation, actions must be taken to either adjust the process parameter or fix the problem [8]. The other metrics (consecutive points on one side of the central line or correlation between points) used to determine when a process becomes out-of-control acts as an early warning to catch a change in the mean before the points begin to fall outside of the control limits [8]. SPC only determines that a change may have occurred. It cannot determine what caused the change or if the signal was due to a special cause. Because of this, a quality engineer must look back at the actual process at the time of the signal to determine what caused the change. In quality control or financial applications, this signal is used to provide predictions to determine when or if a process becomes out-of-control so the problem can either be fixed or make decisions to take advantage of the change.

4. Average Run Length and Type-I Error

The average run length (ARL) is a commonly used metric to determine how quickly a control chart signals to identify that a change may have occurred. There are two types of ARL: the in-control ARL and the out-of-control ARL. The in-control ARL is the expected number of subgroups within an in-control process before it signals that a change may have occurred. Because the process is in-control, any signal is considered a false alarm. Conversely, the out-of-control ARL refers to the expected number of subgroups from an out-of-control process that a control chart takes to signal that a change may have occurred [12].

Average Run Length is the inverse of the probability of type-I error (α).

$$ARL = \frac{1}{\alpha} \tag{II-1}$$

Because of this connection between the ARL and α, we can conclude that as the probability of type-I error decreases, the ARL will increase. The same relationship applies for increases in the probability of type-I error as the ARL decreases [7].

The ARL determines the sensitivity of the control to changes. It does not, however, adjust the charts ability to detect larger or smaller changes. Adjusting the ARL creates a trade off between the amount of time the control chart takes to signal that a change has actually occurred and the probability of type-I error. When the ARL is increased, the probability that a signal actually indicates a change in a parameter increases but the chart will take more time to signal that the change has occurred. When the ARL is decreased, there is a higher probability of type-I error but the signal for an out-of-control process parameter will occur sooner [14].

Because the ARL is correlated to the probability of type-I error, it is easy for an analyst to decide on the best ARL for his statistical endeavors based on the process and user acceptance of type-I error. However, simply determining the best ARL to detect changes in a process will not change the control limits of the chart. The ARL must be used to determine the most appropriate control limits. Fortunately, this study has discovered the correlation between the two values and proposes an analytical method for determining the optimal control limits for the CUSUM control chart.

5. The Cumulative Sum

The CUSUM control chart is one type of control chart that is based on cumulative sums of subgroup averages. The chart is derived from the Sequential Probability Ratio Test (SPRT) [15]. The CUSUM control chart continuously monitors the process for increases in mean with the test,

$$C_t^+ = max(0, Z_t - k + C_{t-1}^+) \tag{II-2}$$

The test statistic, C_t^+, is the current value for the CUSUM, measuring increases in the process. Its value is dependent on the previous statistic value, C_{t-1}^+. The Z_t value is calculated for each data point or subgroup of data points, where $Z_t = x_i - \frac{\mu_0}{\sigma_x}$, and k is an optimality constant [12]. The test statistic is compared to the control limit h^+. h^+ is the control limit used to detect increases in a process parameter. The control charts signals that an increase in the process parameter may have occurred if $C_t^+ > h^+$. Because the C_t^+ statistic is used, it will only display values in which the data point lies above the zero line. The max function allows the control chart to distinguish between values that lie above the zero line and those below it. If the value lies below the line, the statistic is not penalized because the statistic assumes that it is in-control. If $Z_t < k$, C_t^+ will not increase. These values will cause C_t^+ to decrease, but not less than 0 [15].

Conversely, the CUSUM chart can be applied to detect decreases in the process mean by using the test statistic

$$C_t^- = max(0, -Z_t - k + C_{t-1}^-) \tag{II-3}$$

This is compared to the constant h^- where h^- is the control limit used to detect decreases in a process parameter. If $C_t^- > h^-$, then the control chart signals that a decrease in the process parameter may have occurred. The C_t^- statistic and max function applied allow the test to distinguish between values that lie below the zero line and those that lie above it. Similar to C_t^+, the statistic will not be penalized for values on the opposite side of the line, therefore, C_t^- may decrease but it will never obtain a value below 0. Because the statistic is calculated using $-Z_t$, the control chart displays the value below the mean as positive values on the CUSUM control chart [15].

The statistic is cumulative and, therefore, is dependent on previous values. Additionally, because each is cumulative, the statistic is only dependent on its immediate

predecessor. The statistic accounts for this by adding the previous statistic, C^{+}_{t-1}, in the max function calculation. This allows the analyst to easily determine the most likely time when the mean changed because each value is comparable to its predecessor. Additionally, false alarms are easily identifiable because the CUSUM will either decrease or reset when a point returns to below the center line (Table II.1).

Observation	**Z value**	**Test Stat**	**Signal**
37	-1.42379	0	
38	5.585892	5.085892	x
39	-1.72235	2.863544	
40	1.734676	4.098219	
41	-0.34296	3.2552663	
42	-2.30671	0.448554	
43	0.965656	0.914210	
44	-0.88997	0	

Table II.1 False Alarm in the CUSUM control chart

Once the statistic resets to 0, the value will either grow and reset to 0 or remain at 0 as long as the process remains in control. This CUSUM chart was based on an $h^{+} = 4$ and signaled when the test statistic became greater than 4, as seen in Observation 38 in Table II.1. However, it quickly decreased and eventually reset to 0 on Observation 44. When the process is in-control and a false alarm is signaled, the process may not immediately return to 0 and, as a result, the signal must be investigated to determine if a change actually occurred. However, when the parameter changes, the CUSUM will continue to increase because distribution of the parameter has changed (Table II.2).

Based on this, it can be determined that the most likely point where the mean changed in Table II.2 is at observation 113 because that is when the statistic last reset to 0 before exceeding $h^{+} = 4$ on Observation 116. This is the last point when the value of the test statistic was reset and the CUSUM continued to increase. The statistic values continue to increase without resetting to 0 as long as the new mean remains constant for the process. At this time, the Z_t term must be adjusted to account for the change in mean. The mean and standard deviation must be recalculated from a sufficient sample of data within the new mean. At this point, a new CUSUM will be calculated to determine if there is another change in the process mean.

Observation	Z value	Test Stat	Signal
113	-0.46514	0	
114	0.669804	0.169804	
115	4.107976	3.777780	
116	1.874190	5.151971	x
117	0.605128	5.257099	
118	0.268618	5.025717	
119	-0.02588	4.499841	
120	1.471952	5.471793	
121	0.588009	5.559802	
122	1.838617	6.898419	
123	2.657684	9.056104	
124	1.354226	9.910330	
125	1.032596	10.44293	
126	1.682784	11.62571	
127	1.083179	12.20889	

Table II.2 Change in Process Mean

But why use the CUSUM chart? The CUSUM control chart is very good at detecting small changes in the process mean. CUSUM performs better than other control charts at detecting changes smaller than two standard deviations from the mean [12]. The ARL of CUSUM is compared with the ARLs of other control charts in Table II.3. Because the CUSUM has the highest ARL when $\delta = 0$, it is the least sensitive to false alarms and, therefore, is the best chart to use for small changes. However, as Table II.3 shows, as δ increases, the CUSUM becomes more sensitive to false alarms and is not as good at detecting large changes.

Average Run Length (ARL)

$\lvert\delta\rvert$	$\bar{X}$ Chart without WERR	$\bar{X}$ Chart with WERR	CUSUM
0	370.37	91.75	370.40
1	43.86	9.22	9.90
2	6.30	3.13	3.9
3	2.00	1.67	2.5

Table II.3 ARL Performance Comparison between Control Charts

This section only discussed the CUSUM control chart because it is the control chart

that will be implemented in the study. However, it is clear that the CUSUM is much more sensitive to small changes in the mean and much less sensitive toward larger changes [12].

This study is interested in determining the optimal values for k and h based on the user's acceptability of type-I error probability. Recall that type-I error and ARL are inversely related equation II-1. The proposed function will allow the user to determine the best values for the parameters k and h to achieve the desired ARL.

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III. DATA AND METHODOLOGY

A. DATA

The data for this study was generated using C++ code (Appendix A). It utilized a Monte Carlo simulation designed to estimate the ARL of an in-control CUSUM control chart based on the input of the optimality constant and decision interval. Initial data collection comprised of the collection ARL estimates using k over the interval $[0.1, 2]$ and h over the interval $[3, 5]$ (Appendix B). A three-dimensional plot of the data showed a bimodal tendency in the ARL values. Initial attempts to model a surface utilized a third order polynomial function and the sum of squared error was used to find the best correlation for the model.

The bimodal spikes in the data occurred when the ARL values began to grow into very large values. Because ARL is inversely proportional to type-I error, large ARL values create very small probabilities of type-I error, so small that it is basically equal to zero. In normal statistical testing, the acceptable α value is generally between 0.1% and 5%. Therefore, a new data set was created in order to remove the bimodal portions of the surface. Furthermore, the difficulties in modeling a surface lead to the creation of a data set that centered the ARL within a confidence interval of the average sample ARL in order to create a contour plot of the data for several ARL values. The data set uses ARL values of 20, 50, 100, 250, 500, 750, and 1000, limiting the probability of type-I error to the interval $[.001, .05]$. The simulations were run at increasing k values within the interval [0.05, 1.25] increasing in value 0.05 for each entry. The user then adjusted the h value until the ARL value in question fell within the confidence interval produced by the simulation. Each simulation created the confidence interval based on the distribution of ARLs over 100,000 runs (Appendix C).

The data showed that lower k values require larger h values in order to place the desired ARL value within the confidence interval. Very large ARL values with a small optimality constant force the decision interval to be much larger because of the small correction factor applied by the small k value. Furthermore, as the ARL value increase, small changes in k cause larger changes in the h values.

B. METHODOLOGY

Data was collected and plotted in Microsoft Excel in order to fit a function to each individual ARL data set. The graph plotted the dependent h values against the independent k values. The desired end state of the fitted functions would be a contour plot where the desired ARL can be used to determine the optimal k and/or h values.

After the data was plotted, the graph was analyzed to determine trends in the data. Initial analysis suggested that the data exhibited the characteristics of decay. Three functions that could produce curves to fit the data, the exponential (equation III-1), power (equation III-2), and logarithmic functions (equation III-3) were created through regression analysis

$$f(k) = a_1 e^{\lambda k} \tag{III-1}$$

$$f(k) = a_2 k^{b_2} \tag{III-2}$$

$$f(k) = a_3 Ln(k) + b_3 \tag{III-3}$$

The best function for each ARL curve was determined by minimizing the sum of squared error by changing the values of the constants in each equation. After analyzing the SSE and the r^2 values for the three functions, it was determined that the exponential (equation III-1) and logarithmic functions (equation III-3) estimated the decision interval better for the lower ARLs and the power function (equation III-2) estimated the larger values better than the others. In order to create a consistent function that provided accurate estimations for all of the ARL values, a combination of the functions was attempted to determine if a better fit for all of the contours could be found.

Two combinations were created based on the observations of what functions offered the best estimations for ARL. These functions are linear combinations of two functions that account for the fact that one estimation is better for small ARL values and the other is better for large. These functions are termed fading functions. The coefficient, a_1, was altered to account for the increase in ARL levels through the introduction of the k independent value in the coefficient.

The first fading function was the combination of the exponential and power function.

$$f(k) = \left(\frac{1}{ak}\right) e^{\lambda k} + bk^c \tag{III-4}$$

The alteration of the coefficient of the exponential function to $\left(\frac{1}{ak}\right)$ allows the exponential portion of the equation to become insignificant as the optimality constant increases. This was applied because small changes in the k value cause larger changes in the decision interval as the ARL value gets larger. Therefore, the deterioration of k becomes much more significant as the ARL increases. Similar to the regression analysis of the first three functions, this fading function was fit by minimizing the SSE.

A second fading function was also created to combine the logarithmic and power function. The creation of this function was also a linear combination of two functions constructed by replacing the coefficient of the logarithmic function with the value $\left(\frac{1}{ak}\right)$.

$$f(k) = \left(\frac{1}{ak}\right) ln(k) + bk^c \tag{III-5}$$

Both of these fading functions were designed to incorporate the strengths of each of the functions involved in the linear combination to create the best estimate of h for each ARL level.

The contour plots of the ARL levels were created and two products were formed. The first product is two reference tables that allow an analyst to determine the most suitable decision interval for the process modeled using the CUSUM control chart. The tables are very similar to the common statistical tables found in the appendices of most statistics books. One table allows the analyst to use the k value in the CUSUM test statistic and the desired ARL (λ) for the analysis to determine the h value. The other table uses α instead of λ (Appendix D).

The second product is comprised of two surface functions designed to evaluate the control limit (h). One function uses k and α to estimate h and the other uses k and λ. The functions were created by altering the logarithmic and power fading function (equation III-5) to create a function of k and either λ or α.

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IV. RESULTS

This study, focused on developing an analytical method of determining the optimal control limits for a CUSUM control chart based on user accepted type-I error probability, determined that the designation of the parameters for the control chart does not have to be an arbitrary task. This study created two valuable products for analysts desiring to model processes using the CUSUM: reference tables and a surface functions.

The reference tables allow analysts to determine the appropriate control limits for common ARLs or if exact control limits are unnecessary (Appendix D). It provides an easy method for determining the control limits is computational efficiency is valued in the analysis.

The first surface is a function of (λ) and (k).

$$h(\lambda, k) = \left(\frac{\lambda^{0.1}}{5k}\right) ln(k) + (0.53ln(\lambda) + \left(\frac{\pi}{10}\right))k^{-0.89} \tag{IV-1}$$

Equation IV-1 allows the analyst to estimate the most appropriate h value based on his/her accepted λ and k values. The second function (equation IV-2 allows the analyst to use α and k instead of λ

$$h(\alpha, k) = \left(\frac{1}{5k\alpha^{0.1}}\right) ln(k) - (0.53ln(\alpha) + \left(\frac{\pi}{10}\right))k^{-0.89} \tag{IV-2}$$

Both equations IV-1 and IV-2 return the h value. The functions were created assuming that the analyst needs to estimate the control limit. This function provides more than the opportunity to determine the appropriate control limit. It allows analysts the ability to set a control limit based on the specifications of the process with an accepted λ value or α probability and find the correct optimality constant to utilize in the CUSUM test statistic.

Equations IV-1 and IV-2 were created by transforming the constants in the logarithmic and power fading function (equation III-5) into functions of λ. The best fit of the function was determined by minimizing the SSE. After equation IV-1 was modeled, several tests were conducted to determine how accurately the surface function estimated the control limits based on the empirical data. A cross validation of the r^2 values provided the reference value used to determine how well the function performed. To perform the cross validation, 10 percent of the data was removed and the function was evaluated based on the remaining 90 percent of the data. The cross validation was performed ten times, removing

a different 10 percent of the data for each iteration. The cross validation r^2 value is the average of r^2 values calculated for each of the ten iterations. The r^2 value of 0.9907 was compared to the r^2 value for the function and SSE for all of the level curves to determine if the function successfully estimated h. This value scored higher than the other r^2 values of 0.9906 and 0.9879, respectively. Because the cross validation r^2 value outperformed both of the other values it was compared to.

The second surface function (equation IV-2 was created by using equation IV-1 and substituting α for λ. The substitution applied the inverse relationship between the two values (equation II-1). This allowed the creation of another surface function with the same ability to estimate the h value of the CUSUM control chart. Equation IV-2 allows an analyst to set the control limit based on his/her acceptance of type-I error probability instead of using λ.

V. DISCUSSION

We have a reference table and surface function designed to determine the appropriate control limit for the CUSUM. But what can we do with it? The most significant aspect of this study is that we can now apply CUSUM techniques to processes that do not have consequences directly associated with changes in the parameters.

Two examples were discussed earlier in this paper: Al Qaeda and CompanyCommand. Both of these examples illustrate the need for statistical tests in social network analysis. The use of CUSUM control charts is very appropriate when analyzing human behavior because the each entry is dependent on the previous statistic. We can all agree that one action may influence another.

Ian McCulloh's analysis of Al Qaeda in 2007 identified an increase in activity in 1997 [5], however his use of arbitrary parameters did not allow this change to be signaled until between 2000 and 2001, the year Al Qaeda attacked the World Trade Centers. If this research had been done in the 1990's, the test would not have signaled that a change occurred until it was too late to prevent an attack. If he could have used appropriate control limits for his analysis based on his acceptance for type-I error, the chart may have signaled earlier, giving our government time to act and possibly prevent the devastating attacks of September 11th.

The CompanyCommand administrators want to know when an individual becomes more vested in their organization so they could tailor services to help develop their members. It is possible that members of an organization increase their participation level as a result of a significant event or milestone in their lives. The application of statistics could help determine when those significant events occur within a member's life. The investigation of the signal could help determine what an individual values within an organization. This information could be applied to help increase membership in an organization. The information could help organizations like CompanyCommand, which is committed to developing leaders, to help individuals through the significant events and provide better services to facilitate it.

Conversely, insight could help deter members from becoming more vested in an organization that poses a threat. For example, governments could determine what makes a person likely to join a terrorist cell or become more active. That information could be used to create countermeasures in order to decrease participation in terrorist cells that threaten

to destroy our way of life.

The development of this analytical approach to determining the appropriate control limits greatly increases the applications of CUSUM techniques. Effective tests can now be used in situations outside of the business and financial world where loss functions do not necessarily exist. It provides the framework and methodology to the creation of similar functions to estimate the appropriate control limits for other statistical tests that may be values in other disciplines where an analytical method for determining these values does not exist.

APPENDIX A. C++ COMPUTER CODE

```
/***
 *** A quick little program to simulate the performance of
 *** the CUSUM control chart.
 ***
 *** Note that the file "nextseed" must be created PRIOR to using
 *** this code for the first time.
 ***
 *** This code can be compiled with the following command:
 ***     cc sourcefile.c -o executablefile -lm
 *** where "sourcefile.c" is the name that you have given to this C code,
 *** and where "executablefile" is the name you choose for the
 *** the executable code.
 ***
 ***/

#include <stdio.h>
#include <math.h>

main()
{
    unsigned  ix;         /* seed for random number generator           */
    unsigned  ix0;        /* initial seed for random number generator   */
    int     N,            /* Number of independently seeded runs        */
            R,            /* Run length                                 */
            in_control,   /* a 0,1 indicator variable                   */
            count;        /* loop counter                               */
    float   z,            /* random number N(0,1)                       */
            h,            /* decision interval                          */
            k,            /* reference value                            */
            ch,           /* high cusum value                           */
            cl,           /* low cusum value                            */
            d,            /* standardized magnitudes of change in mean  */
            t,            /* change point                               */
            ARL,          /* average run length                         */
            s,            /* sample standard deviation of run lengths   */
            sum,          /* sum of R                                   */
            sum_sq,       /* sum of squared R                           */
            normal();     /* function which returns a N(0,1) r.v.       */

    FILE *fp;             /* File pointer for randon number seeds       */

    printf("Enter optimality constant ");
    printf("(k)   = ");   scanf("%f", &k);

    printf("Enter decision interval ");
    printf("(h)  = ");     scanf("%f", &h);
```

```
if ( (fp = fopen("nextseed", "r")) == NULL ) {
   printf("Can't open: %s\n", "nextseed");
   exit(-1);
}
fscanf(fp, "%d", &ix0);
fclose(fp);
ix=-ix0;
printf("\nSeed to be used to start this run = %ld\n", ix);

/* Set simulation parameters*/
d = 0;
t = 0;
N = 100000;

sum = sum_sq = 0.0e0;
for (count = 1; count <= N; count++)
{
    R=0;             /* initialize the run length to zero */
    in_control = 1;  /* initially assume process is in-control */
    ch = 0; cl = 0;  /* initialize the control chart  */

    do {
        R++;  /* increment run length
              /* generate an Xbar while considering change point */
        if (R < t) z = normal(&ix);
        if (R > t) z = normal(&ix) + d;
                  /* compare appropriate cusum value to decision interval */
        if ((z - k + ch) > 0) ch = (z -k +ch);
        else ch = 0;
        if ((-z -k + cl) > 0) cl = (-z -k +cl);
        else cl = 0;
        if ((ch > h || cl > h) && (R < t)) ch=cl=0;
        if ((ch > h || cl > h) && (R > t)) in_control = 0;
    } while ( in_control );

    /* At this point, cusum value has plotted beyond decision interval */

    sum    += R;            /* increment sum    of run length by R */
    sum_sq += R*R;          /* increment sum_sq of run length by R^2 */
}

ARL= sum/(float) N;
printf("ARL     = %f  (Average Run Length)\n", sum/( (float) N) );

s = sqrt( (sum_sq - ((float) N*ARL*ARL) ) / ((float) N-1.0e0) );
printf("S       = %f  (standard deviation of the run lengths)\n", s );

printf("Approximate 95 percent confidence interval:\n [ %f , %f ]\n\n",
    ARL-2.0e0*s/sqrt(N), ARL+2.0e0*s/sqrt(N));
```

```
    printf("Seed to be used to start the next run  = %ld\n", ix);

    if ( (fp = fopen("nextseed", "w")) == NULL )
    {   printf("Can't open: %s\n", "nextseed");
        exit(-1);
    }
    fprintf(fp, "%d", ix);
    fclose(fp);

}


float normal(ix)
unsigned *ix;
{
    float u, ran1();
    double vnorm();

    u = ran1(ix);
    return( vnorm( (double) u ));
}



#define PLIM 1.0e-18
#define P0 -0.322232431088e0
#define P1 -1.0
#define P2 -0.342242088547e0
#define P3 -0.0204231210245e0
#define P4 -0.453642210148e-4
#define Q0 0.099348462606e0
#define Q1 0.588581570495e0
#define Q2 0.531103462366e0
#define Q3 0.10353775285e0
#define Q4 0.38560700634e-2


double vnorm(phi)
double phi;
{
/*
 *
 * vnorm returns the inverse of the cdf of the normal dist.  it uses a
 * rational approximation which seems to have a relative accuracy of
 * about 5 decimal places.
 *
 * ref: kennedy and gentle, statistical computing, dekker, 1980.
 * code is published in bratley, fox and schrage
 *
 * inputs:
 *     phi = prob.;  0. <= phi <= 1.
```

```
 *
 * outputs:
 *     f inverse of phi, ie, a value such that prob.(x<=vnorn)=phi.
 *
 * Modified 16 November 1987 by Joe Pignatiello
 *    1.  converted code from FORTRAN to C.
 *    2.  no error checking is now done.
 *
 */
double p;
double y;
double vtemp;

/*  p = phi;
if (phi > 0.5)
p = 1.0 - phi;
 */
p = ( phi > 0.5 ? 1.0-phi : phi );
if (p >= PLIM)
{
y = sqrt( -log( p*p ) );
vtemp = y + ( ( ( (y*P4 + P3)*y + P2)*y + P1 )*y + P0) /
( ( ( (y*Q4 + Q3)*y + Q2)*y + Q1)*y + Q0);
}
else
{
vtemp = 8.0;
}
return( phi < 0.5 ? -vtemp : vtemp );
}

/* Uniform  Random number generator - Park and Miller (with Bays-Durham shuffel
with added safeguards)*/

#define IA 16807
#define IM 2147483647
#define AM (1.0/IM)
#define IQ 127773
#define IR 2836
#define NTAB 32
#define NDIV (1+(IM-1)/NTAB)
#define EPS 1.2e-7
#define RNMX (1.0-EPS)

float ran1(long *idum)
{
 int  j;
 long k;
 static long iy=0;
 static long iv[NTAB];
 float temp;
```

```
if(*idum <= 0 || !iy){
    if (-(*idum) < 1) *idum = 1;
    else *idum = -(*idum);
    for (j=NTAB+7;j>=0;j--){
       k=(*idum)/IQ;
       *idum=IA*(*idum-k*IQ)-IR*k;
       if(*idum < 0) *idum+=IM;
       if(j < NTAB) iv[j] = *idum;
      }
      iy = iv[0];
    }

    k=(*idum)/IQ;
    *idum=IA*(*idum-k*IQ)-IR*k;
    if(*idum < 0) *idum += IM;
    j=iy/NDIV;
    iy = iv[j];
    iv[j] = *idum;
    if((temp=AM*iy) > RNMX) return RNMX;
    else return temp;

  }
```

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APPENDIX B. MONTE CARLO SIMULATION DATA

Table B.1: Simulations to estimate ARL (10000 runs)

k	**h**	**ARL**	ln **ARL**	α	$f(x)$	E^2	**ARL**^{2}
0.25	3	58.65418	4.071658843	0.017049083	11991.76173	142399055.7	143802349.3
0.25	3	58.43379	4.067894318	0.017113386	11991.76173	142404315.6	143802349.3
0.25	3	58.91897	4.07616311	0.016972462	11991.76173	142392736.3	143802349.3
0.25	3	58.68879	4.072248738	0.017039029	11991.76173	142398229.7	143802349.3
0.25	3	58.83294	4.074701902	0.016997281	11991.76173	142394789.4	143802349.3
0.25	3.25	76.72502	4.340227861	0.013033558	10551.15412	109713664.9	111326853.2
0.25	3.25	76.84879	4.341839725	0.013012567	10551.15412	109711072.1	111326853.2
0.25	3.25	76.62155	4.338878369	0.013051159	10551.15412	109715832.5	111326853.2
0.25	3.25	76.8271	4.341557442	0.01301624	10551.15412	109711526.5	111326853.2
0.25	3.25	76.72263	4.34019671	0.013033964	10551.15412	109713715	111326853.2
0.25	3.5	99.98077	4.604977867	0.010001923	5384.081645	27921722.06	28988335.16
0.25	3.5	99.08071	4.595934771	0.010092782	5384.081645	27931234.88	28988335.16
0.25	3.5	100.10058	4.606175481	0.009989952	5384.081645	27920455.89	28988335.16
0.25	3.5	99.80417	4.603209966	0.010019621	5384.081645	27923588.43	28988335.16
0.25	3.5	99.87997	4.603969165	0.010012017	5384.081645	27922787.34	28988335.16
0.25	3.75	129.70357	4.865251616	0.007709888	-1513.519135	2700180.858	2290740.172
0.25	3.75	129.50378	4.86371007	0.007721782	-1513.519135	2699524.299	2290740.172
0.25	3.75	129.35662	4.862573086	0.007730567	-1513.519135	2699040.746	2290740.172
0.25	3.75	129.75208	4.865625553	0.007707006	-1513.519135	2700340.286	2290740.172
0.25	3.75	130.20008	4.869072344	0.007680487	-1513.519135	2701812.857	2290740.172
0.25	4	168.07284	5.124397457	0.005949801	-8145.711666	69119012.81	66352618.54
0.25	4	168.6098	5.12758717	0.005930853	-8145.711666	69127941.44	66352618.54
0.25	4	167.03012	5.118174155	0.005986944	-8145.711666	69101676	66352618.54
0.25	4	167.33533	5.119999763	0.005976024	-8145.711666	69106750.35	66352618.54
0.25	4	168.05936	5.12431725	0.005950279	-8145.711666	69118788.67	66352618.54
0.25	4.25	216.43764	5.377302469	0.004620268	-12516.55939	162129213.4	156664259
0.25	4.25	215.31554	5.37210458	0.004644347	-12516.55939	162100639.3	156664259
0.25	4.25	216.02302	5.375384976	0.004629136	-12516.55939	162118654.9	156664259
0.25	4.25	216.42344	5.377236859	0.004620572	-12516.55939	162128851.8	156664259
0.25	4.25	215.32418	5.372144706	0.00464416	-12516.55939	162100859.3	156664259
0.25	4.5	281.66176	5.640706918	0.003550358	-12630.12576	166714256.9	159520076.6
0.25	4.5	281.3282	5.639521959	0.003554567	-12630.12576	166705643.3	159520076.6
0.25	4.5	280.197	5.635492927	0.003568918	-12630.12576	166676433.7	159520076.6
0.25	4.5	278.52244	5.629498631	0.003590375	-12630.12576	166633198.2	159520076.6
0.25	4.5	279.4067	5.632668426	0.003579012	-12630.12576	166656028.2	159520076.6
0.25	4.75	358.80924	5.882790882	0.002786996	-6490.474203	46912683.68	42126255.38

Continued on next page

Table B.1 – continued from previous page

k	h	ARL	ln ARL	α	$f(x)$	E^2	ARL^2
0.25	4.75	362.20448	5.892208914	0.002760871	-6490.474203	46959205.13	42126255.38
0.25	4.75	361.16272	5.889328605	0.002768835	-6490.474203	46944928.52	42126255.38
0.25	4.75	359.96056	5.88599447	0.002778082	-6490.474203	46928456.44	42126255.38
0.25	4.75	359.25904	5.884043688	0.002783507	-6490.474203	46918845.5	42126255.38
0.25	5	465.73624	6.143619465	0.002147138	7898.331824	55243477.12	62383645.61
0.25	5	464.26868	6.140463436	0.002153925	7898.331824	55265294.83	62383645.61
0.25	5	466.16368	6.144536817	0.002145169	7898.331824	55237123.32	62383645.61
0.25	5	466.26512	6.144754399	0.002144703	7898.331824	55235615.5	62383645.61
0.25	5	465.2504	6.142575755	0.00214938	7898.331824	55250699.46	62383645.61
0.5	3	58.4378	4.067962941	0.017112212	-25358.92995	646042583.3	643075328.2
0.5	3	58.6868	4.072214829	0.017039607	-25358.92995	646055241.2	643075328.2
0.5	3	59.6792	4.088983551	0.016756257	-25358.92995	646105691.1	643075328.2
0.5	3	59.6852	4.089084083	0.016754572	-25358.92995	646105996.1	643075328.2
0.5	3	58.6157	4.071002579	0.017060276	-25358.92995	646051626.9	643075328.2
0.5	3.25	76.7917	4.341096561	0.013022241	-6309.534382	40785160.82	39810224.11
0.5	3.25	76.4996	4.337285512	0.013071964	-6309.534382	40781430.01	39810224.11
0.5	3.25	76.7097	4.340028167	0.013036161	-6309.534382	40784113.47	39810224.11
0.5	3.25	77.2812	4.347450718	0.012939758	-6309.534382	40791413.27	39810224.11
0.5	3.25	76.3527	4.335363394	0.013097114	-6309.534382	40779553.82	39810224.11
0.5	3.5	100.7386	4.612529043	0.009926682	4432.621338	18765208.05	19648131.92
0.5	3.5	99.8351	4.603519825	0.010016517	4432.621338	18773036.58	19648131.92
0.5	3.5	99.591	4.601071799	0.010041068	4432.621338	18775151.91	19648131.92
0.5	3.5	100.3871	4.609033713	0.009961439	4432.621338	18768253.49	19648131.92
0.5	3.5	100.2515	4.607682029	0.009974913	4432.621338	18769428.41	19648131.92
0.5	3.75	129.8419	4.866317556	0.007701674	9185.77056	82009843.9	84378380.78
0.5	3.75	128.9436	4.859375099	0.007755329	9185.77056	82026114.59	84378380.78
0.5	3.75	127.9769	4.851849779	0.00781391	9185.77056	82043625.99	84378380.78
0.5	3.75	129.6265	4.864657238	0.007714472	9185.77056	82013745.24	84378380.78
0.5	3.75	130.1453	4.868651519	0.00768372	9185.77056	82004348.85	84378380.78
0.5	4	170.3395	5.137793504	0.005870629	10268.14664	101965709	105434835.4
0.5	4	167.8249	5.122921174	0.005958591	10268.14664	102016499.2	105434835.4
0.5	4	168.0644	5.124347239	0.0059501	10268.14664	102011661.2	105434835.4
0.5	4	167.5637	5.121363577	0.00596788	10268.14664	102021775.7	105434835.4
0.5	4	164.2745	5.101538809	0.006087372	10268.14664	102088232.2	105434835.4
0.5	4.25	214.7639	5.369539285	0.004656276	9997.982921	95711374.41	99959662.48
0.5	4.25	217.191	5.380777151	0.004604242	9997.982921	95663890.6	99959662.48
0.5	4.25	221.6909	5.401284069	0.004510785	9997.982921	95575885.67	99959662.48
0.5	4.25	218.8935	5.38858531	0.004568432	9997.982921	95630589.9	99959662.48
0.5	4.25	215.3898	5.37244941	0.004642745	9997.982921	95699128.16	99959662.48
0.5	4.5	278.1341	5.628103371	0.003595388	10693.51276	108480112.7	114351215.2

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Table B.1 – continued from previous page

k	h	ARL	ln ARL	α	$f(x)$	E^2	ARL^2
0.5	4.5	286.6017	5.658093447	0.003489163	10693.51276	108303797.9	114351215.2
0.5	4.5	280.4969	5.636562673	0.003565102	10693.51276	108430899.3	114351215.2
0.5	4.5	272.7958	5.60872353	0.003665746	10693.51276	108591342	114351215.2
0.5	4.5	282.8132	5.644786609	0.003535903	10693.51276	108382665.4	114351215.2
0.5	4.75	369.9744	5.913433814	0.00270289	14672.96951	204575669.2	215296034.3
0.5	4.75	363.4589	5.895666223	0.002751343	14672.96951	204762094	215296034.3
0.5	4.75	357.6611	5.87958589	0.002795943	14672.96951	204928055	215296034.3
0.5	4.75	364.9911	5.89987297	0.002739793	14672.96951	204718246.3	215296034.3
0.5	4.75	360.8144	5.888363699	0.002771508	14672.96951	204837784	215296034.3
0.5	5	468.9459	6.15048741	0.002132442	24254.58653	565756700	588284967.6
0.5	5	460.9995	6.133396958	0.0021692	24254.58653	566134783.6	588284967.6
0.5	5	462.6032	6.136869667	0.00216168	24254.58653	566058470.6	588284967.6
0.5	5	456.1712	6.122868178	0.002192159	24254.58653	566364572	588284967.6
0.5	5	468.2435	6.14898846	0.002135641	24254.58653	565790114.6	588284967.6
0.75	3	222.5798	5.405285689	0.004492771	-22898.22477	534571604.2	524328697.8
0.75	3	220.7416	5.396992787	0.004530184	-22898.22477	534486606.2	524328697.8
0.75	3	216.7065	5.378543903	0.004614536	-22898.22477	534300047.8	524328697.8
0.75	3	221.6246	5.40098496	0.004512134	-22898.22477	534527435.1	524328697.8
0.75	3	215.7103	5.373936304	0.004635847	-22898.22477	534253994.6	524328697.8
0.75	3.25	320.4341	5.769676639	0.003120766	2940.673178	6865652.828	8647558.742
0.75	3.25	322.4969	5.776093524	0.003100805	2940.673178	6854847.025	8647558.742
0.75	3.25	329.1223	5.796429414	0.003038384	2940.673178	6820197.99	8647558.742
0.75	3.25	318.8438	5.764701328	0.003136332	2940.673178	6873989.289	8647558.742
0.75	3.25	320.0507	5.768479421	0.003124505	2940.673178	6867662.174	8647558.742
0.75	3.5	475.4326	6.164225126	0.002103348	13441.96708	168131016.3	180686478.9
0.75	3.5	469.6351	6.15195601	0.002129313	13441.96708	168281396.9	180686478.9
0.75	3.5	466.678	6.145639512	0.002142805	13441.96708	168358126.6	180686478.9
0.75	3.5	472.4389	6.157908426	0.002116676	13441.96708	168208661.1	180686478.9
0.75	3.5	477.299	6.168143129	0.002095123	13441.96708	168082618.3	180686478.9
0.75	3.75	687.7629	6.533444156	0.001453989	12460.57817	138599179.4	155266008.3
0.75	3.75	686.339	6.531371675	0.001457006	12460.57817	138632708	155266008.3
0.75	3.75	693.2046	6.541325194	0.001442576	12460.57817	138471080.7	155266008.3
0.75	3.75	684.5694	6.528790027	0.001460772	12460.57817	138674382.6	155266008.3
0.75	3.75	691.078	6.538252697	0.001447015	12460.57817	138521134.3	155266008.3
0.75	4	998.7693	6.906523821	0.001001232	3851.427711	8137660.009	14833495.41
0.75	4	998.1082	6.905861687	0.001001895	3851.427711	8141432.231	14833495.41
0.75	4	1016.7396	6.924356316	0.000983536	3851.427711	8035456.686	14833495.41
0.75	4	998.9501	6.906704827	0.001001051	3851.427711	8136628.52	14833495.41
0.75	4	1009.9598	6.917665807	0.000990138	3851.427711	8073939.888	14833495.41
0.75	4.25	1444.156	7.275280347	0.000692446	-8530.563051	99495020.15	72770505.97

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Table B.1 – continued from previous page

k	h	ARL	ln ARL	α	$f(x)$	E^2	ARL^2
0.75	4.25	1483.3401	7.302051648	0.000674154	-8530.563051	100278256.3	72770505.97
0.75	4.25	1466.0476	7.290325351	0.000682106	-8530.563051	99932224.51	72770505.97
0.75	4.25	1460.6721	7.286651951	0.000684616	-8530.563051	99824779.85	72770505.97
0.75	4.25	1443.6167	7.274906841	0.000692705	-8530.563051	99484261.71	72770505.97
0.75	4.5	2116.7134	7.657619882	0.000472431	-20830.47287	526573357.5	433908599.8
0.75	4.5	2160.2716	7.677989234	0.000462905	-20830.47287	528574331.1	433908599.8
0.75	4.5	2108.27	7.653622985	0.000474323	-20830.47287	526185924.2	433908599.8
0.75	4.5	2109.0928	7.654013181	0.000474138	-20830.47287	526223672.9	433908599.8
0.75	4.5	2105.555	7.652334369	0.000474934	-20830.47287	526061374.2	433908599.8
0.75	4.75	3158.7634	8.057935901	0.00031658	-29193.38048	1046661214	852253463.9
0.75	4.75	3105.914	8.041063315	0.000321966	-29193.38048	1043244424	852253463.9
0.75	4.75	3128.7064	8.048374907	0.000319621	-29193.38048	1044717300	852253463.9
0.75	4.75	3084.5404	8.034157946	0.000324197	-29193.38048	1041864176	852253463.9
0.75	4.75	3147.594	8.05439363	0.000317703	-29193.38048	1045938630	852253463.9
0.75	5	4554.7888	8.423934442	0.000219549	-29764.36465	1177804293	885917402.8
0.75	5	4583.4132	8.43019924	0.000218178	-29764.36465	1179769843	885917402.8
0.75	5	4572.392	8.427791761	0.000218704	-29764.36465	1179012857	885917402.8
0.75	5	4460.666	8.403053361	0.000224182	-29764.36465	1171352723	885917402.8
0.75	5	4480.9776	8.407596516	0.000223166	-29764.36465	1172743466	885917402.8
1	3	973.9445	6.88135432	0.001026753	-4856.12696	33989733.23	23581969.06
1	3	968.5003	6.875748793	0.001032524	-4856.12696	33926282.72	23581969.06
1	3	995.851	6.903597648	0.001004166	-4856.12696	34245646.05	23581969.06
1	3	978.1262	6.885638701	0.001022363	-4856.12696	34038509.94	23581969.06
1	3	967.7188	6.874941549	0.001033358	-4856.12696	33917179.44	23581969.06
1	3.25	1607.8183	7.382633446	0.000621961	17347.03356	247722897.1	300919573.4
1	3.25	1626.1651	7.393979822	0.000614944	17347.03356	247145705.2	300919573.4
1	3.25	1625.7428	7.393720098	0.000615103	17347.03356	247158983.2	300919573.4
1	3.25	1585.813	7.368852489	0.000630591	17347.03356	248416073.6	300919573.4
1	3.25	1620.8052	7.390678342	0.000616977	17347.03356	247314258.5	300919573.4
1	3.5	2689.1516	7.896981033	0.000371864	20891.48142	331324810.8	436453995.8
1	3.5	2658.4812	7.885510261	0.000376155	20891.48142	332442296.9	436453995.8
1	3.5	2691.0064	7.897670529	0.000371608	20891.48142	331257290.8	436453995.8
1	3.5	2661.506	7.886647407	0.000375727	20891.48142	332332003.7	436453995.8
1	3.5	2695.258	7.899249212	0.000371022	20891.48142	331102546.6	436453995.8
1	3.75	4426.9632	8.39546912	0.000225888	13831.15392	88438803.02	191300818.6
1	3.75	4348.1252	8.377500043	0.000229984	13831.15392	89927833.62	191300818.6
1	3.75	4399.6376	8.389277453	0.000227291	13831.15392	88953500.01	191300818.6
1	3.75	4404.6136	8.390407816	0.000227035	13831.15392	88859662.32	191300818.6
1	3.75	4494.9152	8.410702081	0.000222474	13831.15392	87165353.36	191300818.6
1	4	7324.8816	8.89903227	0.000136521	4219.988371	9640361.961	17808301.85

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Table B.1 – continued from previous page

k	h	ARL	ln ARL	α	$f(x)$	E^2	ARL^2
1	4	7240.416	8.887433942	0.000138114	4219.988371	9122983.06	17808301.85
1	4	7228.2032	8.885745764	0.000138347	4219.988371	9049356.455	17808301.85
1	4	7225.2952	8.88534337	0.000138403	4219.988371	9031869.134	17808301.85
1	4	7292.176	8.894557272	0.000137133	4219.988371	9438336.826	17808301.85
1	4.25	12044.9008	9.396396679	8.30227E-05	111.9220958	142395980.8	12526.55552
1	4.25	12177.6824	9.407360244	8.21174E-05	111.9220958	145582571.7	12526.55552
1	4.25	12067.6024	9.398279653	8.28665E-05	111.9220958	142938291.5	12526.55552
1	4.25	12100.4552	9.400998351	8.26415E-05	111.9220958	143724926	12526.55552
1	4.25	12004.176	9.393009868	8.33043E-05	111.9220958	141425702.9	12526.55552
1	4.5	19728.2912	9.889808986	5.06886E-05	9560.892401	103375998.3	91410663.51
1	4.5	19693.9808	9.888068325	5.07769E-05	9560.892401	102679480.5	91410663.51
1	4.5	19347.3152	9.870308939	5.16868E-05	9560.892401	95774071.2	91410663.51
1	4.5	19859.7632	9.896451014	5.03531E-05	9560.892401	106066739.7	91410663.51
1	4.5	19534.04	9.879913864	5.11927E-05	9560.892401	99463673.03	91410663.51
1	4.75	32502.6016	10.38907541	3.07668E-05	40620.8366	65905739.52	1650052366
1	4.75	32602.0032	10.39212901	3.0673E-05	40620.8366	64301689.1	1650052366
1	4.75	31727.8638	10.36495056	3.1518E-05	40620.8366	79084965.22	1650052366
1	4.75	32683.8208	10.39463546	3.05962E-05	40620.8366	62996219.81	1650052366
1	4.75	32698	10.39506919	3.05829E-05	40620.8366	62771339.79	1650052366
1	5	53483.056	10.88712017	1.86975E-05	101345.692	2290831925	10270949288
1	5	54067.712	10.89799247	1.84953E-05	101345.692	2235207393	10270949288
1	5	53446.6848	10.88643989	1.87102E-05	101345.692	2294314891	10270949288
1	5	53519.2448	10.88779658	1.86849E-05	101345.692	2287369052	10270949288
1	5	52750.704	10.8733324	1.89571E-05	101345.692	2361472859	10270949288
1.25	3	4724.2588	8.46046596	0.000211673	4537.359277	34931.43152	20587629.21
1.25	3	4802.048	8.476797773	0.000208244	4537.359277	70060.11983	20587629.21
1.25	3	4788.4836	8.473969064	0.000208834	4537.359277	63063.42536	20587629.21
1.25	3	4761.7376	8.468367923	0.000210007	4537.359277	50345.63162	20587629.21
1.25	3	4819.8588	8.480499912	0.000207475	4537.359277	79805.98023	20587629.21
1.25	3.25	9056.6488	9.111254441	5.99826E-05	15954.80353	47584538.74	254555755.8
1.25	3.25	9039.7824	9.109390382	0.000110622	15954.80353	47817517.29	254555755.8
1.25	3.25	9157.4008	9.122317662	0.000109201	15954.80353	46204683.93	254555755.8
1.25	3.25	9021.5	9.107365896	0.000110846	15954.80353	48070697.9	254555755.8
1.25	3.25	9200.9744	9.12706467	0.000108684	15954.80353	45614207.97	254555755.8
1.25	3.5	16671.4976	9.72145581	5.99826E-05	15260.52692	1990838.273	232883681.7
1.25	3.5	16686.0112	9.722325995	5.99304E-05	15260.52692	2032005.445	232883681.7
1.25	3.5	16877.2288	9.733720584	5.92514E-05	15260.52692	2613724.983	232883681.7
1.25	3.5	16823.584	9.73053699	5.94404E-05	15260.52692	2443147.449	232883681.7
1.25	3.5	16875.2656	9.733604255	5.92583E-05	15260.52692	2607381.019	232883681.7
1.25	3.75	31388.4928	10.35419663	3.18588E-05	18817.74802	158023624.4	354107640.4

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Table B.1 – continued from previous page

k	h	ARL	ln ARL	α	$f(x)$	E^2	ARL^2
1.25	3.75	30845.2352	10.33673757	3.24199E-05	18817.74802	144660447.9	354107640.4
1.25	3.75	31210.1472	10.34849855	3.20409E-05	18817.74802	153571557.5	354107640.4
1.25	3.75	31350.0864	10.3529723	3.18978E-05	18817.74802	157059505.3	354107640.4
1.25	3.75	31589.7248	10.36058718	3.16559E-05	18817.74802	163123390.9	354107640.4
1.25	4	58733.9008	10.98077237	1.70259E-05	42989.68543	247880317.5	1848113054
1.25	4	59145.8816	10.98776224	1.69073E-05	42989.68543	261022674.5	1848113054
1.25	4	58834.112	10.9824771	1.69969E-05	42989.68543	251045853.2	1848113054
1.25	4	58904.256	10.98366863	1.69767E-05	42989.68543	253273556.3	1848113054
1.25	4	58499.9936	10.97678192	1.7094E-05	42989.68543	240569659.4	1848113054
1.25	4.25	109171.6992	11.60067714	9.15988E-06	104139.5578	25322447.43	10845047491
1.25	4.25	107722.8672	11.58731716	9.28308E-06	104139.5578	12840106.51	10845047491
1.25	4.25	109013.6576	11.59922845	9.17316E-06	104139.5578	23756849.21	10845047491
1.25	4.25	109274.0864	11.60161456	9.1513E-06	104139.5578	26363384.31	10845047491
1.25	4.25	110506.4064	11.61282877	9.04925E-06	104139.5578	40536761.55	10845047491
1.25	4.5	202258.9568	12.21730412	4.94416E-06	218630.5836	268030164.1	47799332086
1.25	4.5	204527.68	12.2284586	4.88931E-06	218630.5836	198891890	47799332086
1.25	4.5	199076.3648	12.20144377	5.0232E-06	218630.5836	382367472.9	47799332086
1.25	4.5	200935.4368	12.21073893	4.97672E-06	218630.5836	313118220.3	47799332086
1.25	4.5	203035.7276	12.22113724	4.92524E-06	218630.5836	243199533.7	47799332086
1.25	4.75	378054.3488	12.84279324	2.64512E-06	402825.9815	613633788.6	1.62269E+11
1.25	4.75	375870.4384	12.83699978	2.66049E-06	402825.9815	726601306	1.62269E+11
1.25	4.75	374797.3888	12.83414086	2.66811E-06	402825.9815	785602011	1.62269E+11
1.25	4.75	376618.1632	12.83898712	2.65521E-06	402825.9815	686849742.1	1.62269E+11
1.25	4.75	382962.8672	12.85569331	2.61122E-06	402825.9815	394543311.3	1.62269E+11
1.25	5	702061.824	13.46177675	1.42438E-06	673088.9702	839426258.6	4.53049E+11
1.25	5	709644.544	13.47251948	1.40916E-06	673088.9702	1336309977	4.53049E+11
1.25	5	697250.56	13.45490011	1.4342E-06	673088.9702	583782422.7	4.53049E+11
1.25	5	701874.8461	13.46151039	1.42476E-06	673088.9702	828626652.6	4.53049E+11
1.25	5	711835.2896	13.47560183	1.40482E-06	673088.9702	1501277269	4.53049E+11
1.5	3	24859.0064	10.1209754	4.02269E-05	-18947.77027	1919033683	359017998.4
1.5	3	24362.0224	10.10078074	4.10475E-05	-18947.77027	1875738142	359017998.4
1.5	3	24818.816	10.11935735	4.0292E-05	-18947.77027	1915514074	359017998.4
1.5	3	24861.1392	10.12106119	4.02234E-05	-18947.77027	1919220549	359017998.4
1.5	3	24529.64	10.10763746	4.0767E-05	-18947.77027	1890285204	359017998.4
1.5	3.25	52641.0144	10.87125084	1.89966E-05	-22190.76014	5599794481	492429835.6
1.5	3.25	52564.4288	10.86979491	1.90243E-05	-22190.76014	5588338274	492429835.6
1.5	3.25	52577.9872	10.87005282	1.90194E-05	-22190.76014	5590365579	492429835.6
1.5	3.25	52184.7328	10.86254526	1.91627E-05	-22190.76014	5531713950	492429835.6
1.5	3.25	52184.7328	10.86254526	1.91627E-05	-22190.76014	5531713950	492429835.6
1.5	3.5	111864.32	11.62504199	8.9394E-06	-14971.53387	16087333827	224146826.5

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Table B.1 – continued from previous page

k	h	ARL	ln ARL	α	$f(x)$	E^2	ARL^2
1.5	3.5	111527.6928	11.6220282	8.96638E-06	-14971.53387	16002054348	224146826.5
1.5	3.5	113123.6864	11.63623707	8.83988E-06	-14971.53387	16408385456	224146826.5
1.5	3.5	111512.3456	11.62189059	8.96762E-06	-14971.53387	15998171766	224146826.5
1.5	3.5	111882.752	11.62520674	8.93793E-06	-14971.53387	16092009844	224146826.5
1.5	3.75	241207.6032	12.39341327	4.14581E-06	32940.6107	43375140167	1085083833
1.5	3.75	239963.4688	12.38824198	4.1673E-06	32940.6107	42858463778	1085083833
1.5	3.75	238466.8672	12.38198566	4.19345E-06	32940.6107	42241042113	1085083833
1.5	3.75	238241.3824	12.38103965	4.19742E-06	32940.6107	42148406863	1085083833
1.5	4	496035.7376	13.11440325	2.01598E-06	151776.3757	1.18515E+11	23036068226
1.5	4	507392.768	13.13704067	1.97086E-06	151776.3757	1.26463E+11	23036068226
1.5	4	487562.24	13.09717323	2.05102E-06	151776.3757	1.12752E+11	23036068226
1.5	4	497694.976	13.11774267	2.00926E-06	151776.3757	1.1966E+11	23036068226
1.5	4.5	2191071.642	14.59990132	4.56398E-07	723141.5758	2.15482E+12	5.22934E+11
1.5	4.5	2173302.374	14.5917584	4.60129E-07	723141.5758	2.10297E+12	5.22934E+11
1.5	4.5	2176185.549	14.59308415	4.5952E-07	723141.5758	2.11134E+12	5.22934E+11
1.5	4.5	2205579.059	14.60650064	4.53396E-07	723141.5758	2.19762E+12	5.22934E+11
1.5	4.5	2135622.246	14.57426861	4.68248E-07	723141.5758	1.9951E+12	5.22934E+11
1.5	5	9268724.531	16.04215634	1.0789E-07	1940969.684	5.3696E+13	3.76736E+12
2	3	712318.6688	13.47628066	1.40387E-06	-261473.8936	9.48272E+11	68368597037
2	3	704953.856	13.46588763	1.41853E-06	-261473.8936	9.33983E+11	68368597037
2	3	705835.6864	13.46713775	1.41676E-06	-261473.8936	9.35688E+11	68368597037
2	3	700292.7616	13.45925376	1.42797E-06	-261473.8936	9.24995E+11	68368597037
2	3	714854.1952	13.47983388	1.39889E-06	-261473.8936	9.53217E+11	68368597037
2	3.25	2240653.926	14.62227831	4.46298E-07	-292560.8614	6.41718E+12	85591857609
2	3.25	2223204.352	14.61446011	4.49801E-07	-292560.8614	6.32907E+12	85591857609
2	3.25	2185106.842	14.59717528	4.57644E-07	-292560.8614	6.13884E+12	85591857609
2	3.25	2200967.987	14.60440782	4.54346E-07	-292560.8614	6.21769E+12	85591857609
2	3.25	2217175.654	14.61174472	4.51024E-07	-292560.8614	6.29878E+12	85591857609
2	3.5	5793267.302	15.57220699	1.72614E-07	-195321.5241	3.58632E+13	38150497767
2	3.5	5683406.029	15.55306126	1.75951E-07	-195321.5241	3.45594E+13	38150497767
2	3.5	5657136.333	15.54842837	1.76768E-07	-195321.5241	3.42513E+13	38150497767
2	3.5	572639.0544	13.25801087	1.7463E-06	-195321.5241	5.89763E+11	38150497767
2	3.5	5670271.795	15.55074761	1.76358E-07	-195321.5241	3.44052E+13	38150497767
2	3.75	14143392.97	16.46475815	7.07044E-08	110676.1427	1.96917E+14	12249208555
2	4	33887672.73	17.33856187	2.95093E-08	705864.1632	1.10103E+15	4.98244E+11
0.1	3	11.6673	2.456790057	0.085709633	63591.32897	4042373379	4043857121
0.1	3	11.7689	2.465460459	0.084969708	63591.32897	4042360459	4043857121
0.1	3	11.7122	2.460631034	0.085381056	63591.32897	4042367669	4043857121
0.1	3	11.8918	2.475849087	0.084091559	63591.32897	4042344831	4043857121
0.1	3	11.631	2.453673947	0.08597713	63591.32897	4042377995	4043857121

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Table B.1 – continued from previous page

k	h	ARL	ln ARL	α	$f(x)$	E^2	ARL^2
0.1	3.25	13.4335	2.597751587	0.074440764	41917.97051	1755990222	1757116252
0.1	3.25	13.4412	2.598324617	0.074398119	41917.97051	1755989577	1757116252
0.1	3.25	13.2304	2.582517212	0.075583505	41917.97051	1756007244	1757116252
0.1	3.25	13.4674	2.60027195	0.074253382	41917.97051	1755987381	1757116252
0.1	3.25	13.5658	2.60755192	0.073714783	41917.97051	1755979134	1757116252
0.1	3.5	15.1284	2.716573772	0.066100843	15528.72991	240671831.7	241141452.5
0.1	3.5	15.2043	2.721578283	0.065770867	15528.72991	240669476.8	241141452.5
0.1	3.5	15.3621	2.731903437	0.065095267	15528.72991	240664580.7	241141452.5
0.1	3.5	15.2214	2.722702332	0.065696979	15528.72991	240668946.2	241141452.5
0.1	3.5	15.097	2.714496049	0.066238325	15528.72991	240672806	241141452.5
0.1	3.75	17.155	2.842289676	0.058292043	-13793.26844	190727795.7	190254254.4
0.1	3.75	17.1499	2.841992343	0.058309378	-13793.26844	190727654.8	190254254.4
0.1	3.75	17.0483	2.836050492	0.058656875	-13793.26844	190724848.6	190254254.4
0.1	3.75	17.0543	2.836402371	0.058636238	-13793.26844	190725014.3	190254254.4
0.1	3.75	17.4525	2.859482905	0.057298381	-13793.26844	190736013	190254254.4
0.1	4	19.4841	2.969598748	0.0513239	-44264.90015	1961106689	1959381386
0.1	4	19.172	2.953450881	0.052159399	-44264.90015	1961079046	1959381386
0.1	4	19.2312	2.95653396	0.051998835	-44264.90015	1961084290	1959381386
0.1	4	19.3918	2.964850296	0.051568189	-44264.90015	1961098514	1959381386
0.1	4	19.1261	2.951053894	0.052284574	-44264.90015	1961074981	1959381386
0.1	4.25	21.6949	3.07707721	0.046093782	-74103.04083	5494476447	5491260660
0.1	4.25	21.9393	3.088279549	0.045580306	-74103.04083	5494512679	5491260660
0.1	4.25	21.7926	3.081570463	0.045887136	-74103.04083	5494490931	5491260660
0.1	4.25	21.832	3.083376783	0.045804324	-74103.04083	5494496772	5491260660
0.1	4.25	21.7534	3.079770067	0.045969825	-74103.04083	5494485119	5491260660
0.1	4.5	24.3266	3.191570402	0.041107265	-101524.5661	10312177603	10307237516
0.1	4.5	23.9833	3.177357755	0.04169568	-101524.5661	10312107880	10307237516
0.1	4.5	24.4587	3.196985981	0.040885247	-101524.5661	10312204432	10307237516
0.1	4.5	24.3278	3.191619729	0.041105238	-101524.5661	10312177847	10307237516
0.1	4.5	24.5857	3.202164973	0.04067405	-101524.5661	10312230226	10307237516
0.1	4.75	26.7862	3.28788683	0.037332656	-124746.3515	15568335891	15561652213
0.1	4.75	27.0133	3.296329337	0.037018802	-124746.3515	15568392564	15561652213
0.1	4.75	26.9568	3.294235585	0.037096391	-124746.3515	15568378464	15561652213
0.1	4.75	27.2482	3.304987464	0.036699672	-124746.3515	15568451182	15561652213
0.1	4.75	26.6652	3.283359344	0.037502063	-124746.3515	15568305696	15561652213
0.1	5	29.9463	3.399405778	0.033393107	-141985.2727	20168322432	20159817668
0.1	5	30.1215	3.405239202	0.033198878	-141985.2727	20168372194	20159817668
0.1	5	29.9348	3.399021683	0.033405936	-141985.2727	20168319166	20159817668
0.1	5	29.9079	3.39812266	0.033435982	-141985.2727	20168311525	20159817668
0.1	5	29.742	3.392560188	0.033622487	-141985.2727	20168264405	20159817668

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Table B.1 – continued from previous page

k	h	ARL	ln ARL	α	$f(x)$	E^2	ARL^2
0.2	3	16.6005	2.809432815	0.060239149	26371.50805	694581152.2	695456437.1
0.2	3	16.4483	2.800222128	0.060796556	26371.50805	694589174.7	695456437.1
0.2	3	16.5164	2.804353827	0.060545882	26371.50805	694585585.1	695456437.1
0.2	3	16.3215	2.792483257	0.061268878	26371.50805	694595858.3	695456437.1
0.2	3	16.4679	2.801413032	0.060724197	26371.50805	694588141.6	695456437.1
0.2	3.25	19.0079	2.944854682	0.052609704	18900.61675	356515152.7	357233313.5
0.2	3.25	19.2032	2.955076932	0.052074654	18900.61675	356507777.6	357233313.5
0.2	3.25	19.4216	2.966385849	0.051489064	18900.61675	356499530.3	357233313.5
0.2	3.25	18.905	2.939426437	0.052896059	18900.61675	356519038.6	357233313.5
0.2	3.25	19.4216	2.966385849	0.051489064	18900.61675	356499530.3	357233313.5
0.2	3.5	22.4915	3.11313746	0.044461241	7783.486527	60233043.81	60582662.51
0.2	3.5	22.8058	3.12701489	0.043848495	7783.486527	60228165.35	60582662.51
0.2	3.5	22.3603	3.107287065	0.044722119	7783.486527	60235080.31	60582662.51
0.2	3.5	22.5896	3.117489623	0.044268159	7783.486527	60231521.11	60582662.51
0.2	3.5	22.7129	3.122933045	0.044027843	7783.486527	60229607.29	60582662.51
0.2	3.75	25.8398	3.251915939	0.03869999	-5030.096944	25562496.35	25301875.26
0.2	3.75	25.9098	3.254621276	0.038595435	-5030.096944	25563204.19	25301875.26
0.2	3.75	25.7087	3.246829456	0.038897338	-5030.096944	25561170.7	25301875.26
0.2	3.75	25.8071	3.250649648	0.038749026	-5030.096944	25562165.7	25301875.26
0.2	3.75	26.4343	3.274662409	0.037829638	-5030.096944	25568508.22	25301875.26
0.2	4	30.1081	3.404794239	0.033213653	-17590.34799	310480473	309420342.5
0.2	4	30.005	3.401364034	0.033327779	-17590.34799	310476839.6	309420342.5
0.2	4	29.8966	3.397744762	0.03344862	-17590.34799	310473019.5	309420342.5
0.2	4	30.5018	3.417785698	0.03278495	-17590.34799	310494347.5	309420342.5
0.2	4	30.3226	3.411893309	0.032978702	-17590.34799	310488032.2	309420342.5
0.2	4.25	34.6932	3.546543702	0.028824092	-27947.48096	783002070.5	781061691.8
0.2	4.25	35.0477	3.556709991	0.028532543	-27947.48096	783021910	781061691.8
0.2	4.25	34.6037	3.543960613	0.028898644	-27947.48096	782997061.7	781061691.8
0.2	4.25	34.756	3.548352219	0.028772011	-27947.48096	783005585	781061691.8
0.2	4.25	35.1235	3.558870422	0.028470967	-27947.48096	783026152.1	781061691.8
0.2	4.5	39.2911	3.67099803	0.025451056	-34151.71016	1169024567	1166339307
0.2	4.5	29.9948	3.401024033	0.033339112	-34151.71016	1168388954	1166339307
0.2	4.5	40.392	3.698631746	0.024757378	-34151.71016	1169099850	1166339307
0.2	4.5	39.9178	3.68682234	0.025051481	-34151.71016	1169067423	1166339307
0.2	4.5	39.8339	3.684718309	0.025104245	-34151.71016	1169061685	1166339307
0.2	4.75	45.5498	3.818806233	0.021953993	-34253.24994	1176407664	1173285131
0.2	4.75	45.1635	3.810289238	0.022141774	-34253.24994	1176381165	1173285131
0.2	4.75	45.2604	3.812432478	0.022094369	-34253.24994	1176387812	1173285131
0.2	4.75	45.4518	3.816652424	0.022001329	-34253.24994	1176400941	1173285131
0.2	4.75	45.2331	3.81182912	0.022107704	-34253.24994	1176385939	1173285131

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Table B.1 – continued from previous page

k	h	ARL	ln ARL	α	$f(x)$	E^2	ARL^2
0.2	5	52.3482	3.957917553	0.019102854	-26302.31463	694568252.7	691811754.7
0.2	5	51.3289	3.938253946	0.019482202	-26302.31463	694514527.1	691811754.7
0.2	5	51.4585	3.940775658	0.019433135	-26302.31463	694521358	691811754.7
0.2	5	51.8453	3.948264284	0.019288151	-26302.31463	694541745.4	691811754.7
0.2	5	51.562	3.942784967	0.019394127	-26302.31463	694526813.3	691811754.7
0.4	3	36.41	3.594843462	0.027464982	-16552.90114	275205244	273998536.1
0.4	3	37.0559	3.612427583	0.026986256	-16552.90114	275226674.5	273998536.1
0.4	3	36.5299	3.598131103	0.027374835	-16552.90114	275209222.1	273998536.1
0.4	3	36.7054	3.602923883	0.027243948	-16552.90114	275215045.1	273998536.1
0.4	3	37.246	3.617544557	0.026848521	-16552.90114	275232982	273998536.1
0.4	3.25	46.2283	3.833592165	0.021631771	-3872.03213	15352764.8	14992632.82
0.4	3.25	46.4955	3.839355534	0.021507458	-3872.03213	15354858.79	14992632.82
0.4	3.25	46.1213	3.831274882	0.021681956	-3872.03213	15351926.3	14992632.82
0.4	3.25	46.1552	3.832009631	0.021666031	-3872.03213	15352191.96	14992632.82
0.4	3.25	46.4303	3.837952263	0.02153766	-3872.03213	15354347.82	14992632.82
0.4	3.5	57.1221	4.045191082	0.017506359	2972.753038	8500903.769	8837260.627
0.4	3.5	57.8215	4.057360679	0.017294605	2972.753038	8496825.873	8837260.627
0.4	3.5	56.915	4.041558927	0.017570061	2972.753038	8502111.466	8837260.627
0.4	3.5	57.0766	4.044394225	0.017520315	2972.753038	8501169.093	8837260.627
0.4	3.5	58.5291	4.069524066	0.017085518	2972.753038	8492701.163	8837260.627
0.4	3.75	72.3729	4.28183192	0.013817327	6106.126545	36406183.05	37284781.38
0.4	3.75	71.9712	4.276266039	0.013894447	6106.126545	36411030.72	37284781.38
0.4	3.75	70.7258	4.258810429	0.014139112	6106.126545	36426062.15	37284781.38
0.4	3.75	71.7205	4.27277662	0.013943015	6106.126545	36414056.31	37284781.38
0.4	3.75	72.285	4.280616639	0.013834129	6106.126545	36407243.79	37284781.38
0.4	4	88.8293	4.486716551	0.011257547	7652.760565	57213056.19	58564744.27
0.4	4	89.7687	4.497236362	0.01113974	7652.760565	57198845.95	58564744.27
0.4	4	88.2338	4.47999011	0.011333525	7652.760565	57222065.18	58564744.27
0.4	4	89.8326	4.497947938	0.011131816	7652.760565	57197879.41	58564744.27
0.4	4	88.6503	4.484699417	0.011280278	7652.760565	57215764.11	58564744.27
0.4	4.25	110.2554	4.702799493	0.009069851	9737.327276	92680512.91	94815542.49
0.4	4.25	110.5193	4.705190166	0.009048193	9737.327276	92675431.82	94815542.49
0.4	4.25	109.6317	4.697126566	0.009121449	9737.327276	92692522.11	94815542.49
0.4	4.25	108.6686	4.688302884	0.00920229	9737.327276	92711067.91	94815542.49
0.4	4.25	109.9114	4.699674587	0.009098237	9737.327276	92687136.46	94815542.49
0.4	4.5	136.5391	4.91661102	0.007323909	14484.49885	205863949.1	209800707.1
0.4	4.5	136.0409	4.912955576	0.007350731	14484.49885	205878245.7	209800707.1
0.4	4.5	136.6802	4.917643891	0.007316349	14484.49885	205859900.1	209800707.1
0.4	4.5	135.1997	4.906752945	0.007396466	14484.49885	205902386.2	209800707.1
0.4	4.5	135.6393	4.909999157	0.007372495	14484.49885	205889770.5	209800707.1

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Table B.1 – continued from previous page

k	h	ARL	$\ln$ ARL	α	$f(x)$	E^2	ARL^2
0.4	4.75	169.2837	5.131576006	0.005907243	24018.94748	568806462.2	576909837.9
0.4	4.75	169.9059	5.135244754	0.005885611	24018.94748	568776784.1	576909837.9
0.4	4.75	167.2801	5.119669653	0.005977997	24018.94748	568902036.6	576909837.9
0.4	4.75	169.0899	5.130430526	0.005914014	24018.94748	568815706.4	576909837.9
0.4	4.75	167.4574	5.120728991	0.005971668	24018.94748	568893578.9	576909837.9
0.4	5	205.2456	5.324207311	0.004872212	40465.34532	1620875629	1637444172
0.4	5	203.0382	5.313394139	0.004925182	40465.34532	1621053374	1637444172
0.4	5	206.802	5.331761814	0.004835543	40465.34532	1620750310	1637444172
0.4	5	208.0518	5.337787087	0.004806495	40465.34532	1620649681	1637444172
0.4	5	204.8252	5.322156932	0.004882212	40465.34532	1620909480	1637444172

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APPENDIX C. EXAMPLE SIMULATION & CONFIDENCE INTERVALS

Simulations: ARL = 20			
k (Optimality Constant)	h (Decision Interval)	CI Low	CI High
0.05	4.56	19.9544	20.1213
0.1	4.07	19.9796	20.1625
0.15	3.66	19.8695	20.0615
0.2	3.31	19.9171	20.1192
0.25	3.02	19.9243	20.1347
0.3	2.76	19.8849	20.0991
0.35	2.55	19.9873	20.2089
0.4	2.36	19.9638	20.1886
0.45	2.18	19.8471	20.0731
0.5	2.03	19.8906	20.1185
0.55	1.9	19.9593	20.1914
0.6	1.769	19.8356	20.0682
0.65	1.66	19.8718	20.1074
0.7	1.56	19.9438	20.1814
0.75	1.46	19.8807	20.1197
0.8	1.372	19.866	20.106
0.85	1.29	19.8666	20.1081
0.9	1.212	19.9178	20.1583
0.95	1.135	19.843	20.0842
1	1.065	19.8784	20.1219
1.05	0.998	19.9421	20.1857
1.1	0.935	19.9195	20.1649
1.15	0.871	19.8731	20.1178
1.2	0.81	19.9349	20.1807
1.25	0.75	19.8714	20.1163

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APPENDIX D. CUSUM DECISION INTERVAL REFERENCE TABLES

This chart uses estimated *h* values directly from the empirical data

DECISION INTERVAL (*h*) REFERENCE CHART

k (Optimality Constant)	Average Run Length (λ)						
	20	**50**	**100**	**250**	**500**	**750**	**1000**
0.05	4.560	7.420	10.290	15.200	19.750	22.740	24.940
0.1	4.070	6.370	8.530	11.880	14.770	16.540	17.860
0.15	3.660	5.570	7.250	9.810	11.870	13.120	14.030
0.2	3.310	4.930	6.320	8.350	9.960	10.930	11.620
0.25	3.020	4.420	5.600	7.270	8.590	9.370	9.940
0.3	2.760	4.000	5.020	6.430	7.550	8.210	8.680
0.35	2.550	3.640	4.540	5.780	6.730	7.310	7.710
0.4	2.360	3.340	4.140	5.230	6.080	6.575	6.925
0.45	2.180	3.078	3.797	4.776	5.532	5.977	6.295
0.5	2.030	2.850	3.502	4.385	5.070	5.470	5.755
0.55	1.900	2.650	3.247	4.060	4.677	5.045	5.300
0.6	1.769	2.465	3.020	3.770	4.335	4.668	4.910
0.65	1.660	2.310	2.820	3.512	4.040	4.350	4.565
0.7	1.560	2.170	2.645	3.283	3.772	4.062	4.267
0.75	1.460	2.036	2.640	3.080	3.537	3.808	3.997
0.8	1.372	1.920	2.333	2.895	3.327	3.578	3.760
0.85	1.290	1.810	2.203	2.735	3.140	3.375	3.544
0.9	1.212	1.710	2.084	2.582	2.965	3.188	3.351
0.95	1.135	1.620	1.975	2.447	2.810	3.022	3.173
1	1.065	1.532	1.875	2.323	2.665	2.865	3.010
1.05	0.998	1.450	1.778	2.210	2.534	2.727	2.860
1.1	0.935	1.375	1.690	2.105	2.414	2.596	2.724
1.15	0.871	1.300	1.610	2.006	2.305	2.476	2.602
1.2	0.810	1.230	1.532	1.916	2.201	2.368	2.485
1.25	0.750	1.165	1.457	1.830	2.105	2.265	2.379

This chart uses estimated h values directly from the empirical data

DECISION INTERVAL (h) REFERENCE CHART

		Probability of Type-I Error (α)						
		0.05	**0.02**	**0.01**	**0.004**	**0.002**	**0.0013**	**0.001**
k (Optimality Constant)	**0.05**	4.560	7.420	10.290	15.200	19.750	22.740	24.940
	0.1	4.070	6.370	8.530	11.880	14.770	16.540	17.860
	0.15	3.660	5.570	7.250	9.810	11.870	13.120	14.030
	0.2	3.310	4.930	6.320	8.350	9.960	10.930	11.620
	0.25	3.020	4.420	5.600	7.270	8.590	9.370	9.940
	0.3	2.760	4.000	5.020	6.430	7.550	8.210	8.680
	0.35	2.550	3.640	4.540	5.780	6.730	7.310	7.710
	0.4	2.360	3.340	4.140	5.230	6.080	6.575	6.925
	0.45	2.180	3.078	3.797	4.776	5.532	5.977	6.295
	0.5	2.030	2.850	3.502	4.385	5.070	5.470	5.755
	0.55	1.900	2.650	3.247	4.060	4.677	5.045	5.300
	0.6	1.769	2.465	3.020	3.770	4.335	4.668	4.910
	0.65	1.660	2.310	2.820	3.512	4.040	4.350	4.565
	0.7	1.560	2.170	2.645	3.283	3.772	4.062	4.267
	0.75	1.460	2.036	2.640	3.080	3.537	3.808	3.997
	0.8	1.372	1.920	2.333	2.895	3.327	3.578	3.760
	0.85	1.290	1.810	2.203	2.735	3.140	3.375	3.544
	0.9	1.212	1.710	2.084	2.582	2.965	3.188	3.351
	0.95	1.135	1.620	1.975	2.447	2.810	3.022	3.173
	1	1.065	1.532	1.875	2.323	2.665	2.865	3.010
	1.05	0.998	1.450	1.778	2.210	2.534	2.727	2.860
	1.1	0.935	1.375	1.690	2.105	2.414	2.596	2.724
	1.15	0.871	1.300	1.610	2.006	2.305	2.476	2.602
	1.2	0.810	1.230	1.532	1.916	2.201	2.368	2.485
	1.25	0.750	1.165	1.457	1.830	2.105	2.265	2.379

APPENDIX E. SAMPLE CONSTRUCTION OF SURFACE FUNCTIONS

					Fading Function (Log and Power): f(x) = (1/(ax))Ln(x)+bx^c				b1	b2	b3	b4	b5		
	Simulations: ARL = 20				ARL = 20				5	0.1	0.531852	0.314159	-0.89		0.987865
k (Optimality Constant)	h (Decision Interval)	CI Low	CI High	ARL	f(x)	E^2	a =	0.976336297	Fading Fn		SSE	SST	R^2		
0.05	4.56	19.9544	20.1213	20.03785	4.482832	0.005955	b =	1.180867502	2.243581	5.365797	25.76642	2751.598	0.990636		0.008329
0.1	4.07	19.9796	20.1625	20.07105	4.360549	0.084419	c =	-1.092238571	3.727947	0.117					0.009113
0.15	3.66	19.8695	20.0615	19.9655	3.752057	0.008474	SSE =	0.485165102	3.503996	0.024337					0.009617
0.2	3.31	19.9171	20.1192	20.01815	3.269697	0.001624	SST =	27.57077424	3.188167	0.014843				Cross-Val R^2	0.010096
0.25	3.02	19.9243	20.1347	20.0295	2.901166	0.014122	r^2 =	0.982402921	2.898926	0.014659				0.990683	0.010505
0.3	2.76	19.8849	20.0991	19.992	2.613377	0.021498			2.651295	0.011817					0.010714
0.35	2.55	19.9873	20.2089	20.0981	2.382722	0.027982			2.452861	0.009436					0.011026
0.4	2.36	19.9638	20.1886	20.0762	2.193563	0.027701			2.277383	0.006826					0.011197
0.45	2.18	19.8471	20.0731	19.9601	2.035406	0.020907			2.122566	0.003299			5		0.011323
0.5	2.03	19.8906	20.1185	20.00455	1.901012	0.016638			1.996563	0.001118			0.1		0.011392
0.55	1.9	19.9593	20.1914	20.07535	1.785243	0.013169			1.887631	0.000153			0.53		0.011561
0.6	1.769	19.8356	20.0682	19.9519	1.684356	0.007165			1.783678	0.000215			0.32		0.011658
0.65	1.66	19.8718	20.1074	19.9896	1.595558	0.004153			1.69757	0.001412			-0.9		0.011786
0.7	1.56	19.9438	20.1814	20.0626	1.516724	0.001873			1.621758	0.003814					0.011843
0.75	1.46	19.8807	20.1197	20.0002	1.446206	0.00019			1.548873	0.007898					0.01195
0.8	1.372	19.866	20.106	19.986	1.382707	0.000115			1.48442	0.012638					0.012008
0.85	1.29	19.8666	20.1081	19.98735	1.325189	0.001238			1.42621	0.018553					0.012083
0.9	1.212	19.9178	20.1583	20.03805	1.272814	0.003698			1.374389	0.02637					0.012002
0.95	1.135	19.843	20.0842	19.9636	1.224896	0.008081			1.323292	0.035454					0.012082
1	1.065	19.8784	20.1219	20.00015	1.180868	0.013425			1.279131	0.045852					0.012175
1.05	0.998	19.9421	20.1857	20.0639	1.140255	0.020237			1.23894	0.058052					0.012141
1.1	0.935	19.9195	20.1649	20.0422	1.102661	0.02811			1.199515	0.069968					0.012244
1.15	0.871	19.8731	20.1178	19.99545	1.067749	0.03871			1.162205	0.084801					0.012238
1.2	0.81	19.9349	20.1807	20.0578	1.035229	0.050728			1.12985	0.102304					0.012255
1.25	0.75	19.8714	20.1163	19.99385	1.004856	0.064951			1.096768	0.120248					0.012249

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