Compressed Sensing

Dr. Edward Fuselier

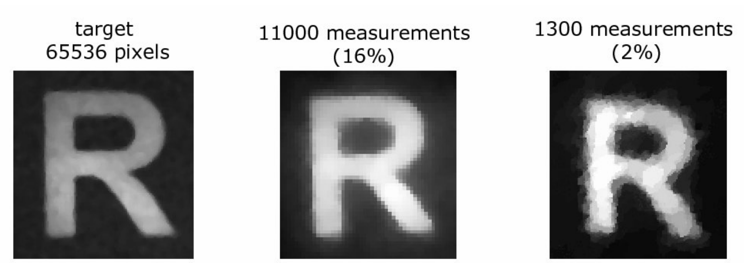
Compressed sensing is an exciting and *brand new* area of mathematics! Read on to get an idea of what compressed sensing is. There is an idea for a student project at the end.

Before we can discuss compressed sensing, we must have a basic idea of what "compression" is. Consider the following example from digital photography. When you take a picture, millions of little sensors are exposed to light. Each sensor then records a color and saves it as a single pixel within the picture. At this point, the information that represents the photo is huge; saving the picture in its present form would take up way too much precious memory. Instead, most digital cameras convert the photograph to a compressed `.jpeg` file, and **the result is sometimes 80% to 90% smaller than the original "raw" photo, and looks just as good!** The mathematics behind compression processes is quite beautiful and surprisingly simple.

With this type of compression, we begin with a huge amount of data and are able compress it into something much smaller, and we can do this without losing the essential information contained in the data. However, consider the following:

- What if we had only a few sensors at our disposal? This is the case for many scientific instruments.
- If we are going to throw away 90% of the information, why waste all that energy acquiring it in the first place?
- Instead of taking millions of measurements, is there a way to take fewer measurements and still get a good representation of the original picture?
- Further, can we find such a method that works well for *any* picture?

Surprisingly, the answer is yes! Enter the mathematics of compressed sensing. To show that this is indeed possible, below is an example from a digital camera that uses compressed sensing and has only **one pixel**! Note we can still see the "R" quite clearly even if we only make 1300 measurements (instead of the full $65,536$).



My (rough) idea for a student project is as follows. For most of the first semester, you can study the "older" compression techniques. I will guide you in learning the basics of Fourier analysis and Wavelets. We can work through examples using these techniques in `MATLAB` (there are built-in toolboxes we can use for some of this). Once you understand these, you can move on to compressed sensing. The end goal would be that you are able to explain some of the mathematical aspects of these methods and provide a few computational examples illustrating how they work.