Homework 4

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Question 1

We set up the transition matrix and check the long term behavior of this system.

$$M = \begin{pmatrix} 1 & 1/2 & 0 & 0 & 0 \\ 0 & 1/4 & 1/3 & 0 & 0 \\ 0 & 1/4 & 1/3 & 1/6 & 0 \\ 0 & 0 & 1/3 & 1/3 & 0 \\ 0 & 0 & 0 & 1/2 & 1. \end{pmatrix}$$

$$\left\{\left\{1, \frac{1}{2}, 0, 0, 0\right\}, \left\{0, \frac{1}{4}, \frac{1}{3}, 0, 0\right\}, \left\{0, \frac{1}{4}, \frac{1}{3}, \frac{1}{6}, 0\right\}, \left\{0, 0, \frac{1}{3}, \frac{1}{3}, 0\right\}, \left\{0, 0, 0, \frac{1}{2}, 1.\right\}\right\}$$

```
p[n_] := M.p[n - 1]
```

```
p[0] = {0, 0, 1, 0, 0}
```

{0, 0, 1, 0, 0}

```
p[100]
```

$\{0.470588, 1.2764\times10^{-17}, 1.66142\times10^{-17}, 1.57983\times10^{-17}, 0.529412\}$

So, we are expecting to win 53% of the time and lose 47% of the time.

$$L = \begin{pmatrix} 1 & 1/3 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 0 \\ 0 & 1/3 & 1/3 & 1/3 & 0 \\ 0 & 0 & 1/3 & 1/3 & 0 \\ 0 & 0 & 0 & 1/3 & 1. \end{pmatrix}$$

$$\left\{\left\{1, \frac{1}{3}, 0, 0, 0\right\}, \left\{0, \frac{1}{3}, \frac{1}{3}, 0, 0\right\}, \left\{0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0\right\}, \left\{0, 0, \frac{1}{3}, \frac{1}{3}, 0\right\}, \left\{0, 0, 0, \frac{1}{3}, 1.\right\}\right\}$$

```
q[n_] := L.q[n - 1]
```

```
q[0] = {0, 0, 1, 0, 0}
```

{0, 0, 1, 0, 0}

```
q[100]
```

$\{0.5, 1.29986\times10^{-10}, 1.83829\times10^{-10}, 1.29986\times10^{-10}, 0.5\}$

In a traditional game of Rock, Paper Scissors we expect to only win 50% of the time. Therefore, I would like to play with the percentages on the sheet, because we are more likely to win.

Question 2

$$MF = \begin{pmatrix} .75 & .1 \\ .2 & .8 \end{pmatrix}$$

{{0.75, 0.1}, {0.2, 0.8}}

```
b = {7, 12}
```

{7, 12}

```
r[n_] := MF.r[n - 1] + b
```

```
r[0] = {106, 120}
```

{106, 120}

```
Table[r[n], {n, 0, 100}]
```

```
{{106, 120}, {98.5, 129.2}, {93.795, 135.06}, {90.8523, 138.807}, {89.0199, 141.216},
 {87.8865, 142.777}, {87.1926, 143.799}, {86.7743, 144.478}, {86.5285, 144.937},
 {86.39, 145.255}, {86.3181, 145.482}, {86.2868, 145.649}, {86.28, 145.777}, {86.2877, 145.877},
 {86.3035, 145.96}, {86.3236, 146.028}, {86.3455, 146.087}, {86.3679, 146.139},
 {86.3898, 146.185}, {86.4108, 146.226}, {86.4307, 146.263}, {86.4493, 146.296},
 {86.4666, 146.327}, {86.4827, 146.355}, {86.4975, 146.38}, {86.5112, 146.404},
 {86.5238, 146.425}, {86.5353, 146.445}, {86.546, 146.463}, {86.5558, 146.48}, {86.5648, 146.495},
 {86.5731, 146.509}, {86.5807, 146.522}, {86.5877, 146.534}, {86.5941, 146.544},
 {86.6, 146.554}, {86.6055, 146.563}, {86.6104, 146.572}, {86.615, 146.58}, {86.6192, 146.587},
 {86.6231, 146.593}, {86.6266, 146.599}, {86.6299, 146.605}, {86.6329, 146.61},
 {86.6356, 146.614}, {86.6382, 146.619}, {86.6405, 146.623}, {86.6426, 146.626},
 {86.6446, 146.629}, {86.6464, 146.632}, {86.648, 146.635}, {86.6495, 146.638}, {86.6509, 146.64},
 {86.6522, 146.642}, {86.6534, 146.644}, {86.6545, 146.646}, {86.6555, 146.648},
 {86.6564, 146.649}, {86.6572, 146.651}, {86.658, 146.652}, {86.6587, 146.653},
 {86.6593, 146.654}, {86.6599, 146.655}, {86.6605, 146.656}, {86.661, 146.657},
 {86.6614, 146.658}, {86.6619, 146.659}, {86.6623, 146.659}, {86.6626, 146.66},
 {86.6629, 146.66}, {86.6633, 146.661}, {86.6635, 146.661}, {86.6638, 146.662},
 {86.664, 146.662}, {86.6642, 146.663}, {86.6644, 146.663}, {86.6646, 146.663},
 {86.6648, 146.663}, {86.6649, 146.664}, {86.6651, 146.664}, {86.6652, 146.664},
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 {86.6657, 146.665}, {86.6658, 146.665}, {86.6659, 146.665}, {86.6659, 146.665},
 {86.666, 146.666}, {86.666, 146.666}, {86.6661, 146.666}, {86.6661, 146.666},
 {86.6662, 146.666}, {86.6662, 146.666}, {86.6663, 146.666}, {86.6663, 146.666},
 {86.6663, 146.666}, {86.6663, 146.666}, {86.6664, 146.666}, {86.6664, 146.666}}
```

Yes, the system does reach equilibrium with about 86 million MySpace customers and about 146 million Facebook customers.

$$\text{Equilbrm} = \text{Inverse}\left[\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \text{MF}\right)\right].\text{b}$$

```
{86.6667, 146.667}
```

No, the equilibrium is found from the recursion equation and does not depend on the initial condition. So, there will be no change in the long term behavior.

Question 3

$$\text{FF} = \begin{pmatrix} .8 & .1 & .2 & .05 \\ .1 & .7 & .2 & .1 \\ 0 & .1 & .5 & 0 \\ .1 & .1 & .1 & .85 \end{pmatrix}$$

```
{{0.8, 0.1, 0.2, 0.05}, {0.1, 0.7, 0.2, 0.1}, {0, 0.1, 0.5, 0}, {0.1, 0.1, 0.1, 0.85}}
```

```
{{l1, l2, l3, l4}, {v1, v2, v3, v4}} = Eigensystem[FF]
```

```
{{1., 0.75, 0.661803, 0.438197}, {{-0.508169, -0.470527, -0.0941054, -0.715201},
  {-0.707107, -1.08264×10^-16, -1.65917×10^-16, 0.707107},
  {0.809017, -0.5, -0.309017, 1.08961×10^-16}, {0.309017, 0.5, -0.809017, 1.26221×10^-16}}}
```

```
FastFood = Solve[c1 * v1 + c2 * v2 + c3 * v3 + c4 * v4 ==
   {4246, 4234, 3490, 0}, {c1, c2, c3, c4}]
```

```
{{c1 → -6694.62, c2 → -6771.25, c3 → -4126.84, c4 → -1958.84}}
```

We find the analytic solution as FF[n]=λ1^n*c1*v1+λ2^n*c2*v2+λ3^n*c3*v3+λ4^n*c4*v4. Since λ1=1 and all others are less than 1, when we take the limit as n-> Infinity, we will only be left with c1*v1.

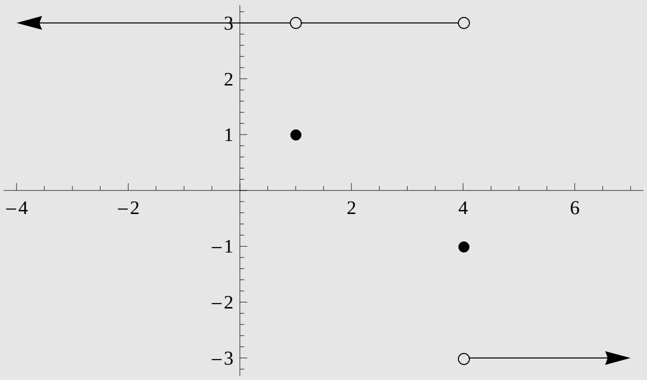
```
c1 * v1 /. FastFood
```

```
{{3402., 3150., 630., 4788.}}
```

Yes, Taco Bell will be the superior fast food chain in the area.

Question 4

```
Graphics[{Arrow[{{.9, 3}, {-4, 3}}], Circle[{1, 3}, .1],
  Disk[{1, 1}, .1], Line[{{1.1, 3}, {3.9, 3}}],
  Circle[{4, 3}, .1], Disk[{4, -1}, .1], Circle[{4, -3}, .1],
  Arrow[{{4.1, -3}, {7, -3}}]}, Axes → True]
```



Question 5

```
f[x_] = Log[x + x^2]
```

$\text{Log}[x + x^2]$

```
x * Log[x + x^2]
```

$x\,\text{Log}[x + x^2]$

```
f[1.]
f[0.5]
f[.1]
f[.05]
f[0.01]
f[0.005]
f[0.001]
```

0.693147

-0.287682

-2.20727

-2.94694

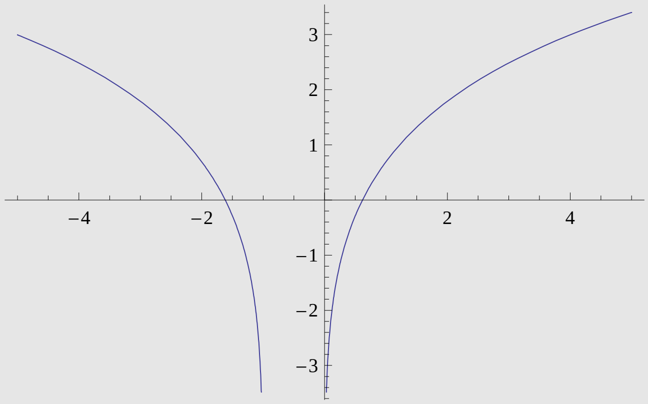
-4.59522

-5.29333

-6.90676

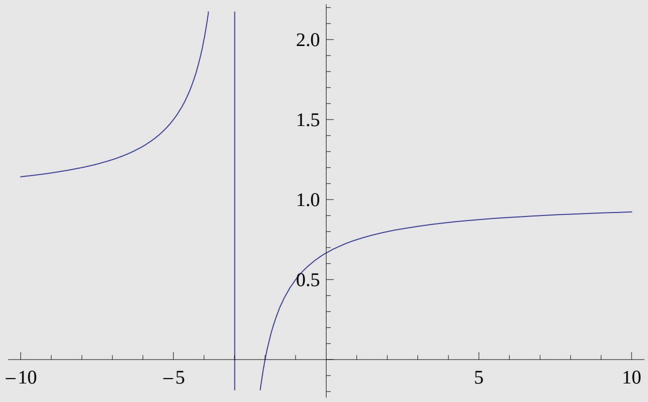
It appears that the limit should be zero. Let's confirm it with a graph.

```
Plot[f[x], {x, -5, 5}]
```



Question 6

```
Plot[(x + 2) / (x + 3), {x, -10, 10}]
```



Question 7

```
g[x_] = 10 x - 1.86 x^2
```

$10\,x - 1.86\,x^2$

```
h[x_] = (g[x] - g[1]) / (x - 1)
```

$$\frac{-8.14 + 10\,x - 1.86\,x^2}{-1 + x}$$

```
h[2]
h[1.15]
h[1.1]
h[1.01]
h[1.001]
h[1.0001]
```

```
4.42
```

```
6.001
```

```
6.094
```

```
6.2614
```

```
6.27814
```

```
6.27981
```