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Section: B22

Quiz 3 — 15 points — 20 Nov 08

INSTRUCTIONS: You will have 15 minutes to complete this quiz. Technology is forbidden on this quiz. Show all work for full credit.

1. Find the **eigenvalues** of

$$\begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}.$$

$\det(M - \lambda \cdot I) = 0$

$\det\left(\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0$

$\det \begin{bmatrix} 1-\lambda & 3 \\ 4 & 5-\lambda \end{bmatrix} = 0$

$(1-\lambda)(5-\lambda) - 12 = 0$

$5 + -\lambda + -5\lambda + \lambda^2 - 12$

$\lambda^2 - 6\lambda - 7 = 0$

$(\lambda - 7)(\lambda + 1)$

eigenvalues $= \lambda = 7 / \lambda = -1$

+

2. The matrix

$$\begin{pmatrix} 7 & 3 \\ 1 & 5 \end{pmatrix}$$

has eigenvalues

$$\lambda_1 = 8 \quad \text{and} \quad \lambda_2 = 4.$$

Find an **eigenvector** associated to the eigenvalue

$$\lambda_2 = 4.$$

+

$(M - \lambda \cdot I) \cdot \vec{v} = \vec{0}$

$\left(\begin{bmatrix} 7 & 3 \\ 1 & 5 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\right) \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$3x + 3y = 0$
$x + y = 0$

$\left[\begin{array}{cc|c} 3 & 3 & 0 \\ \cancel{1} & \cancel{1} & 0 \end{array}\right] = \begin{matrix} x \\ y \end{matrix}$ Row Reduce

$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $x + y = 0 / x = -y$ — Infinitely many solutions

$\vec{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 2 \\ -2 \end{bmatrix} \begin{bmatrix} 3 \\ -3 \end{bmatrix} \ldots$