MA104 - Differential Calculus
Lesson 3: Limit of a Function/Approximating Limits I

1. What does it mean for a function to have a limit?

2. What is meant by each of the following notations?

   a. $\lim_{x \to a} f(x) = L$ b. $\lim_{x \to a^-} f(x) = L$ c. $\lim_{x \to a^+} f(x) = L$

3. Explain in your own words what is meant by the equation

$$\lim_{x \to 2} f(x) = 5.$$

   Is it possible for this statement to be true and yet $f(2) \neq 5$?

Problems

1. For the function below, find $\lim_{x \to 0} f(x)$ by filling out the table in Microsoft Excel. Then, graph the function in Mathematica.

$$f(x) = \frac{\cos(x) - 1}{x}$$

x	$f(x)$
.2	
.1	
.01	
.001	
	Undefined
-.001	
-.01	
-.1	
-.2	

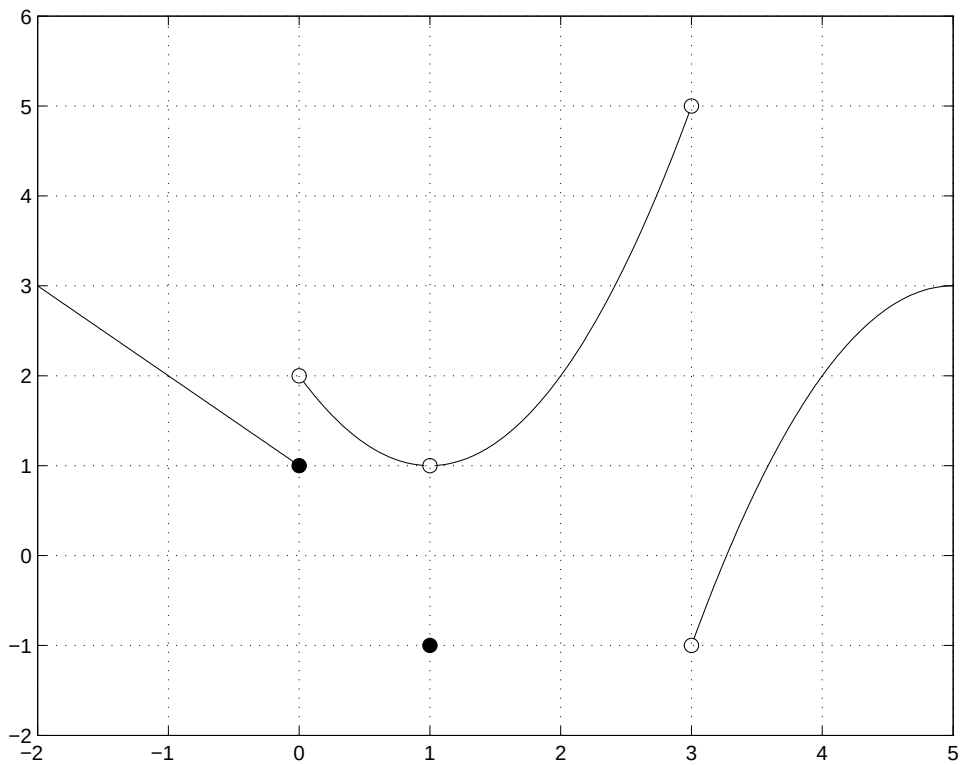
2. For the function below, find the left-hand and right-hand limits of $f(x)$ as x approaches 0. Does $\lim_{x \to 0} f(x)$ exist? If not, explain why. Then, graph the function in Mathematica.

$$f(x) = \frac{1}{1 + e^{1/x}}.$$

x	$f(x)$
.2	
.1	
.01	
.001	
	Undefined
-.001	
-.01	
-.1	
-.2	

3. For the function whose graph is shown, state the value of each quantity, if it exists. If it does not exist, explain why.

a. $\lim_{x \to 0^-} f(x)$ b. $\lim_{x \to 0^+} f(x)$ c. $\lim_{x \to 0} f(x)$

d. $f(1)$ e. $\lim_{x \to 1} f(x)$ f. $\lim_{x \to 3} f(x)$

g. $f(3)$ h. $\lim_{x \to 3^-} f(x)$ i. $\lim_{x \to 3^+} f(x)$