MA104 - Differential Calculus
Lesson 26: Derivatives and the Shapes of Curves

1. The Mean Value Theorem: Let f be a function that satisfies the following hypotheses:

   (a) f is _____ on the closed interval $[a, b]$.

   (b) f is _____ on the open interval (a, b). Then there is a number c in (a, b) such that

   $$f'(c) = \frac{f(b) - f(a)}{b - a}$$

   or equivalently, $f(b) - f(a) = f'(c)(b - a)$.

2. The First Derivative Test: Suppose that c is a critical number of a continuous function f.

   (a) If f' changes from positive to negative at c, then f has a local maximum at c.

   (b) If f' changes from negative to positive at c, then f has a local maximum at c.

   (c) If f' does not change sign at c, then f has no local maximum or minimum at c.

3. The Second Derivative Test: Suppose that $f''(x) > 0$ is continuous near c.

   (a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

   (b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.
   NOTE: This is a result of application of the concavity test on page 300 of text.

4. A point P on a curve $y = f(x)$ is called an inflection point if f is continuous there and the curve changes from concave up to concave down or from concave down to concave up at the point P. How could you find this?

5. Here is a good process to learn about a function:

   (a) Take the derivative of the function. Set equal to zero and solve for independent variable.

   (b) Take the second derivative of the function. Set equal to zero and solve for independent variable.

   (c) Local min and max are located where derivative is equal to zero.

   (d) Use second derivative to discover information on concavity and inflection points.

   (e) Let's try it with $g(t) = t^3 - 3t^2 + 12$

Problems

1. Verify that the function satisfies the hypotheses of the MEAN VALUE THEOREM on the given interval. Then find all numbers c that satisfy the conclusion of the MEAN VALUE THEOREM using the endpoints provided.

   (a) $f(x) = x^3 + x - 1, [0, 2]$

   (b) $f(x) = \dfrac{x}{x+2}, [1, 4]$

2. Suppose you are given a formula for a function f.

   (a) How do you determine where f is increasing or decreasing?

   (b) How do you determine the local minimum or local maximum points?

   (c) How do you determine where the graph of f is concave up or concave down?

   (d) How do you find the inflection points?

3. Do #8 from your textbook on page 304.

4. For the following functions do the following

   (a) Find the intervals on which f in increasing or decreasing.

   (b) Find the local maximum and minimum values of f.

   (c) Find the intervals of concavity and the inflection points.

   (a) $f(x) = 5 - 3x^2 + x^3$

   (b) $f(x) = \dfrac{x^2}{x^2+3}$

   (c) $f(x) = xe^x$

5. Graph the curve $y = x^4 - 4x^3$ without use of technology. Check yourself after wards.