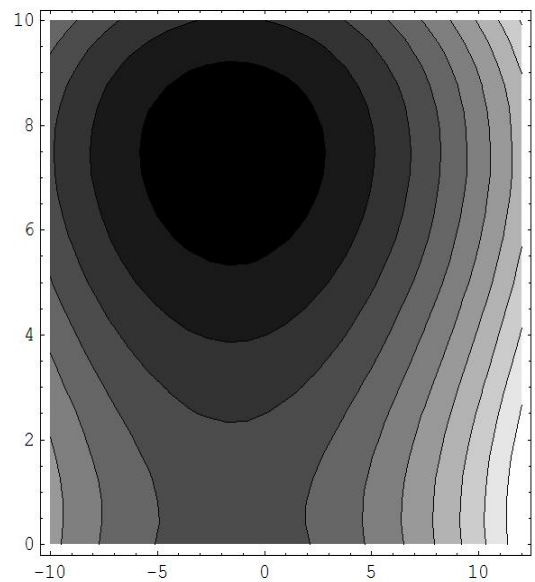
MA104 - Differential Calculus
Lesson 55: Problem Solving Lab

Problems

1. Let $f(x, y) = 6x + 2x^2 + 12y - 12y^2 + y^3 - 250$. The domain of this function is given as $-10 \leq x \leq 12$ and $0 \leq y \leq 10$. The contour plot of this function is shown to the below right.



(a) Find f_x

(b) Find f_y

(c) Find f_{xx}

(d) Find f_{xy}

(e) Find f_{yy}

(f) Calculate $\nabla f|_{(2,2)}$.

(g) Let $\mathbf{v} = \langle -4, -4 \rangle$. Calculate the rate of change of $f(x, y)$ in the direction of $\mathbf{v}$ at the point $(2, 2)$

(h) Which direction gives you the maximum rate of increase of $f(x, y)$ at the point $(-2, 3)$.

(i) What is the maximum rate of change of $f(x, y)$ at the point $(-2, 3)$?

(j) Find and classify the critical points of $f(x, y)$.

(k) What is the maximum value for this function and where does it occur?

2. The temperature at a point (x, y) on a metal plate is given by:
   $T(x, y) = 400e^{\frac{-x^2+y}{2}}$
   From the point (3,5) in which direction is there no chage in temperature?

3. An industrial container is made by placing hemispherical ends on a cylindrical tank. Tyhe container must hold 1000 liters of fluid. What are the dimensions of the cylinder that minimize the material used to make the container.

4. Find all the critical points of $2x^3 + xy^2 + 5x^2 + y^2$. Using the second derivative test, classify each.

5. Find three positive numbers whose sum is 300 and whose product is a maximum.

6. A heated room is shaped like a rectangular box and has a volume of 1000 cubic feet. Because warm air rises the heat loss through the ceiling is five times as great as the heat loss through the floor. The heat loss through the four walls is three times as great as the heat loss through floor. Determine the room dimensions that will minimize the heat loss.