MA 104 Homework # 4

Show all work for full credit. **Due: 21 April 2008**

When taking partial derivatives, show all steps by hand.

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1. Find the domain of the function $f(x,y) = \ln(16-x^2-y^2)+\ln(x^2+y^2-1)$.

2. Describe the graph of the function $f(x,y) = \sqrt{25-x^2-y^2}$ include the domain and range of the function.

   - Determine if the limit exists or not. Explain how you came to this conclusion in words.
   - If the limit does not exist, then find two paths that have different limits when you approach $(0,0)$.
   - If the limit does exist, show that it will work for all paths. This will be similar to the way we described in class and/or on the notes from the limit lesson.

3. $$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2}$$

4. $$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+y^4}$$

5. Show that
$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & if(x,y) \neq (0,0) \\ 0 & if(x,y) = (0,0) \end{cases}$$
has $f_x(0,0) = f_y(0,0) = 0$ but $f(x,y)$ is discontinuous at $(0,0)$. Hence, the existence of the first partial derivatives does not ensure continuity.

6. If $f(x,y) = xy^2 + x^2y - y^3$, show that $xf_x + yf_y = 3f$.

7. If $f(x,y) = \cos 3x \sin 4y$, find $f_x(\pi/12, \pi/6)$ and $f_y(\pi/12, \pi/6)$.

8. Find the derivative of $z = x \ln y$ at the point $(1,2)$ in the direction making an angle of $30°$ with the positive x-axis.

9. If the electrical potential V at any point (x,y) is given by $V = \ln\sqrt{x^2+y^2}$.

   (a) Find the rate of change of V at $(3,4)$ in the direction toward the point $(2,6)$.

   (b) In what direction from $(3,4)$ is the rate of change a maximum?

   (c) What is the magnitude of the maximum rate of change?

10. If the temperature is given by $f(x,y,z) = 3x^2 - 5y^2 + 2z^2$ and you are located at $(\frac{1}{3}, \frac{1}{5}, \frac{1}{2})$ and want to get cool as soon as possible, in which direction should you set out? (Direction is given as a unit vector.)