MA 104 Homework # 5

Show all work for full credit. **Due: 5 May 2008**

If submitted by 2 May 2008, then guaranteed to be returned on Monday 5 May 2008.

When taking partial derivatives, show all steps by hand.

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1. Find relative minima and maxima of the function $f(x, y) = xy(2x+4y+1)$ and classify them.

2. If f is a continuous function of one variable with two relative maxima on a given interval, there must be a relative minimum between the maxima. By considering the functon

$$f(x, y) = 4x^2e^y - 2x^4 - e^{4y}$$

show that it is possible for a continous function of two variables to have only two relative maxima and no relative minima.

3. If a continuous function of one variable has only one critical number on an interval, then a relative extremum must also be an absolute extremum. Show that this result does not extend to functions of two variables by considering the function

$$f(x, y) = 3xe^y - x^3 - e^{3y}$$

In particular, show that it has exactly one critical point, which corresponds to a relative maximum. Is this also an absolute maximum? Explain.

4. A particle of mass m in a rectangular box with dimensions x, y, z has *ground state energy*

$$E(x, y, z) = \frac{k^2}{8m}\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right)$$

where k is a physical constant. If the volume of the box is fixed at 100 cubic feet, find the values of x, y, and z that minimize the ground state energy.

5. Let R be the triangular region in the xy-plane with vertices $(-1, -2)$, $(-1, 2)$ and $(3, 2)$. A plate in the shape of R is heated so that the temperature at (x, y) is

$$T(x, y) = 2x^2 - xy + y^2 - 2y + 1$$

(in degrees Celsius). At what point in R or on its boundary is T maximized? Where is T minimized? What are the extreme temperatures?