Name _____ Section A, C, D 10

1. Given $f(x, y) = \sqrt{\frac{y}{x}}$.

   (a) Find the domain of $f(x, y)$.

   (b) Find the range of $f(x, y)$.

   (c) Find $f_x(x, y)$.

Given $f(x, y) = \sqrt{\frac{y}{x}}$.

(a) Find the domain of $f(x, y)$.
All values in the first and third quadrants and the x-axis.

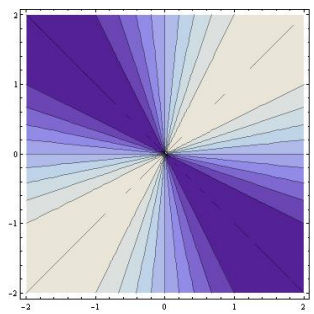
(b) Find the range of $f(x, y)$.
$z \geq 0$

(c) Find $f_x(x, y)$.
$f_x(x, y) = \frac{-\sqrt{y}}{2\sqrt{x^3}}$.

2. Given the countour plot of a function $f(x, y)$. Find

$$\lim_{(x,y)\to(0,0)} f(x, y)$$

if it exists. Explain your answer.



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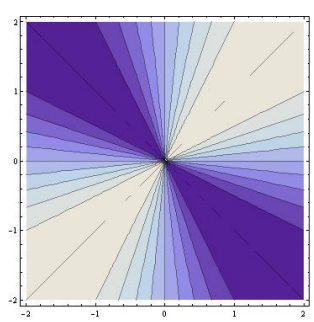

The limit does not exist. As you approach the origin from $y = x$ the limit will be positive and from $y = -x$ will be negative.

3. A store carries two competing brands of bottled water, one from California and the other from upstate New York. To model this situation, assume the owner of the store can obtain both at a cost of \$2 per bottle. Also assume that if the California water is sold for x dollars per bottle and the New York water for y dollars per bottle, then consumers will buy approximately $40 - 50x + 40y$ bottles of California water and $20 + 60x - 70y$ bottles of the New York water each day. How should the owner price the bottled water to generate the largest possible profit?

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Maximize profit: $P(x, y) = [x(40 - 50x + 40y) + y(20 + 60x - 70y)] - [2(40 - 50x + 40y) + 2(20 + 60x - 70y)]$

He should sell the California water at \$2.70 and the New York at \$2.50 giving a total profit of \$7.00 per day.

Find the absolute extrema of $f(x, y) = 2x^2 - y^2$ in the disk $x^2 + y^2 \leq 1$.

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At the point $(1, 0)$ the minimum value is -1 and at $(0, 1)$ and $(0, -1)$ the maximum value of 2.

4. Use Lagrange multipliers to find the distance from the origin to the plane $2x - 3y + z = 4$.

Minimize Distance to origin: $d(x, y) = \sqrt{x^2 + y^2 + z^2}$ subject to the contraint $2x - 3y + z = 4$.

The closest point is at $(\frac{4}{7}, \frac{-6}{7}, \frac{2}{7})$ giving a distance of $2\sqrt{\frac{2}{7}}$ or 1.06904.

5. Find an equation of the plane that contains the point $(2, 1, -1)$ and is orthogonal to the line

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We find the normal vector to be $\langle 3, 5, 2 \rangle$ and passes through $(2, 1, -1)$, so the plane is $3x + 5y + 2 = 9$.

6. The length of a rectangle of constant area 800 square millimeters is increasing at the rate of 4 millimeters per second. What is the width of the rectangle at the moment the width is decreasing at the rate of 0.5 millimeters per second?

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Given $lw = 800$, so differentiating we get $l\frac{dw}{dt} + w\frac{dl}{dt} = 0$.

We know $\frac{dl}{dt} = 4, \frac{dw}{dt} = -.5$, so $4w = .5l$, but $lw = 800$ so $4w = 0.5(\frac{800}{w}) = \frac{400}{w}$. Therefore, the width is 10 mm.