Lesson 28: Row Reduction

There are five parts to each step in this problem. For each step, there is a verbal description, an equation operation, a row operation, a system of equations, and an augmented matrix. At least one of these is provided for you. Provide the remainder.

$$\begin{aligned} 4x_1 - 3x_2 &= 3 \\ 2x_1 + x_2 &= 2 \end{aligned} \qquad \left[\begin{array}{cc|c} 4 & -3 & 3 \\ 2 & 1 & 2 \end{array}\right]$$

Begin by switching equation/row 1 with equation/row 2.

$$E_1 \leftrightarrow E_2 \qquad R_1 \leftrightarrow R_2$$

$$\begin{aligned} 2x_1 + x_2 &= 2 \\ 4x_1 - 3x_2 &= 3 \end{aligned}$$

Multiply the first equation/row by 1/2.

$$\frac{1}{2}E_1 \to E_1 \qquad \frac{1}{2}R_1 \to R_1$$

$$\left[\begin{array}{cc|c} 1 & \frac{1}{2} & 1 \\ 4 & -3 & 3 \end{array}\right]$$

Description:

$$E_2 - 4E_1 \to E_2$$

$$\begin{aligned} 1x_1 + \frac{1}{2}x_2 &= 1 \\ 0x_1 - 5x_2 &= -1 \end{aligned} \qquad \left[\begin{array}{cc|c} 1 & \frac{1}{2} & 1 \\ 0 & -5 & -1 \end{array}\right]$$

Multiply the second row/equation by $-1/5$.

$$1/5R_2 \to R_2$$

$$\begin{aligned} 1x_1 + \frac{1}{2}x_2 &= 1 \\ 0x_1 + x_2 &= 1/5 \end{aligned}$$

Subtract 1/2 times the second row/equation from the first row/equation

$$R_1 - 1/2R_2 \to R_1$$

$$\left[\begin{array}{cc|c} 1 & 0 & 9/10 \\ 0 & 1 & 1/5 \end{array}\right]$$

This system has a unique solution of $x_1 =$_____ and $x_2 =$_____.

Solve the following systems of equations using row reduction:

$$\begin{aligned} x + y &= 5 \\ 3x + 3y &= 10 \end{aligned}$$

$$\begin{aligned} x + 2y &= 5 \\ 2x + 4y &= 10 \end{aligned}$$

$$\left[\begin{array}{rr|r} 1 & 2 & 8 \\ 3 & -4 & 4 \end{array}\right]$$

$$\begin{aligned} 2x + 3y - z &= 6 \\ 2x - y + 2z &= -8 \\ 3x - y + z &= -7 \end{aligned}$$