MA 103
Quiz 7
Friday, October 19, 2007

Directions: You will have 15 minutes to complete this quiz. This is a closed note, open technology, open calculator quiz worth 25 points.

1. The system of equations

$$\begin{array}{rcrcrcr} 3x & - & 2y & + & 8z & = & -7 \\ 1x & + & 1y & + & 1z & = & 6 \\ -2x & + & 1y & - & 5z & = & 3 \end{array}$$

reduces to

$$\left[\begin{array}{rrr|r} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

via row reduction. How many solutions does this system have? Give as many as possible, up to three.

2. The system of equations

$$\begin{array}{rcrcrcrcr} 1x & + & 2y & - & 1z & + & 15w & = & -1 \\ -1x & + & 1y & + & 1z & + & 3w & = & -3 \\ 2x & - & 3y & - & 2z & - & 12w & = & 3 \\ -2x & - & 1y & + & 2z & - & 12w & = & -3 \end{array}$$

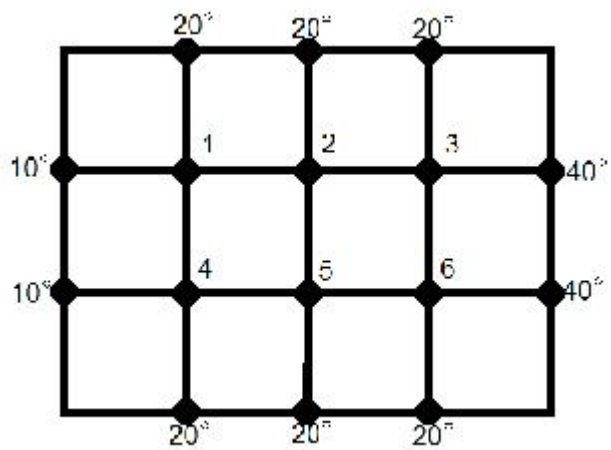
reduces to

$$\left[\begin{array}{rrrr|r} 1 & 0 & -1 & 3 & 2 \\ 0 & 1 & 0 & 6 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

via row reduction. How many solutions does this system have? Give as many as possible, up to three.

3. An important concern in the study of heat transfer is to determine the steady-state temperature distribution of a thin plate when the temperature around the boundary is known. Assume that the grid in the figure represents a cross section of a metal beam, with neligible heat flow in the direction perpendicular to the plate. Let $T_1, T_2, T_3, T_4, T_5, T_6$ denote the temperatures at the six interior nodes of the grid in the figure. The temperature at a node is approximately equal to the average of the four nearest nodes—to the left, above, to the right, and below. For instance,

$$T_1 = (10 + 20 + T_2 + T_4)/4$$



(a) Write a system of six equations whose solution gives estimates for the temperatures $T_1, T_2, T_3, T_4, T_5, T_6$. Present this system as an augmented matrix.

(b) Solve this system.