MA 103 Problem Solving Lab
*Extension Problem**
Solving a System of Supply and Demand Equations

- Let's assume that the Corps of Cadets were just recently granted the priviledge of having cell phones at West Point. Further, assume that the approved phones are the 8 Gb iPhone© , the MOTOKRZR© , and the Blackberry© Pearl 8100. Let the vector $\vec{p}_n = \begin{bmatrix} PI_n \\ PM_n \\ PB_n \end{bmatrix}$ be the prices for the iPhone (PI_n), MOTOKRZR (PM_n), and Blackberry (PB_n) after n months.

- According to the current retail offers on the internet, the price for the iPhone, MOTOKRZR, and Blackberry phones are \$399.00 [1], \$199.99 [2], \$378.99 [3], respectively. The supply and demand functions for the system of these three products in the cadet market is as follows:

$$S(\vec{p}_n) = U.\vec{p}_n = \begin{bmatrix} 2.75 & 0 & 0 \\ 0 & 11.25 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} PI_n \\ PM_n \\ PB_n \end{bmatrix} \text{ for } n \in \{0, 1, 2, \ldots\}$$

$$D(\vec{p}_n) = V.\vec{p}_n + \vec{a} = \begin{bmatrix} -70 & 5 & 65 \\ 5 & -10 & 5 \\ 65 & 7 & -72 \end{bmatrix} \begin{bmatrix} PI_n \\ PM_n \\ PB_n \end{bmatrix} + \begin{bmatrix} 1500 \\ 1000 \\ 1500 \end{bmatrix} \text{ for } n \in \{0, 1, 2, \ldots\}$$

with the current price vector

$$\vec{p}_0 = \begin{bmatrix} 399.00 \\ 199.99 \\ 378.99 \end{bmatrix}.$$

- The recursion equation for this supply and demand system (with volatility constant, $k = 0.008$) is as follows:

$$\vec{p}_n = \vec{p}_{n-1} + k[D(\vec{p}_{n-1}) - S(\vec{p}_{n-1})].$$

[1] http://store.apple.com, accessed on 29 November 2007.
[2] http://www.store.motorola.com, accessed on 29 November 2007.
[3] http://www.cellhut.com, accessed on 29 November 2007.

- Preforming some vector and matrix operations from Block II we see that

$$\vec{p}_n = \vec{p}_{n-1} + k[D(\vec{p}_{n-1}) - S(\vec{p}_{n-1})]$$

$$\vec{p}_n = \vec{p}_{n-1} + k[V.\vec{p}_{n-1} + \vec{a} - U.\vec{p}_{n-1}]$$

$$\vec{p}_n = \vec{p}_{n-1} + kV.\vec{p}_{n-1} + k\vec{a} - kU.\vec{p}_{n-1}$$

$$\vec{p}_n = \vec{p}_{n-1} + kV.\vec{p}_{n-1} - kU.\vec{p}_{n-1} + k\vec{a}$$

$$\vec{p}_n = (I + kV - kU).\vec{p}_{n-1} + k\vec{a}.$$

  So this is a system of (nonhomogeneous) linear recursion equations, with matrix $M = I + kV - kU$ and vector $\vec{b} = k\vec{a}$.

- Use the recursion equation to compute the price at the end of the year. What happens if you try to compute the price in one year and three months?

- The closed form solution for a system of linear recursion equations mirrors the closed form solution for linear recursion equations:

$$\vec{p}_n = M^n(\vec{p}_0 - \vec{p}_*) + \vec{p}_*$$

  or

$$\vec{p}_n = c_1\lambda_1^n\vec{v}_1 + c_2\lambda_2^n\vec{v}_2 + c_3\lambda_3^n\vec{v}_3 + \vec{p}_*$$

  where $c_1, c_2,$ and c_3 are claculated from the vector decomposition of

$$\vec{p}_0 - \vec{p}_*.$$

- Calculate the equilibrium vector, $\vec{p}_*$ for this system by setting $D(\vec{p}_*) = S(\vec{p}_*)$ and solving for $\vec{p}_*$.

- Decompose the vector $\mathbf{\vec{p}_0} - \mathbf{\vec{p}}_*$ into a linear combination of eigenvectors. Use the decomposition to get the general equation

$$\vec{p}_n = c_1\lambda_1^n\vec{v}_1 + c_2\lambda_2^n\vec{v}_2 + c_3\lambda_3^n\vec{v}_3 + \vec{p}_*.$$

- Use the closed form solution to compute the price for the next 10 years.

- If these administrative rules were never changed for any future generations of cadets, what prices would you expect for the 8 Gb iPhone© , the MOTOKRZR© , and the Blackberry© Pearl 8100?