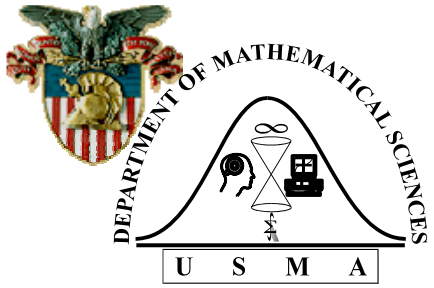
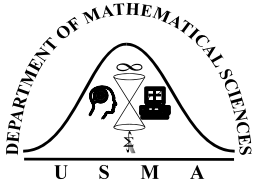


Lesson 8 – The Fundamental Theorem of Calculus II

Courage is the first of human qualities because it is the quality that guarantees all others.

Sir Winston Churchill



Objectives

- **Evaluate integrals using the Fundamental Theorem of Calculus.**
- **Given an initial condition, and any rate of change function find the particular function that satisfies the initial condition.**
- **Given initial conditions and an equation for acceleration find velocity and position equations.**

Review

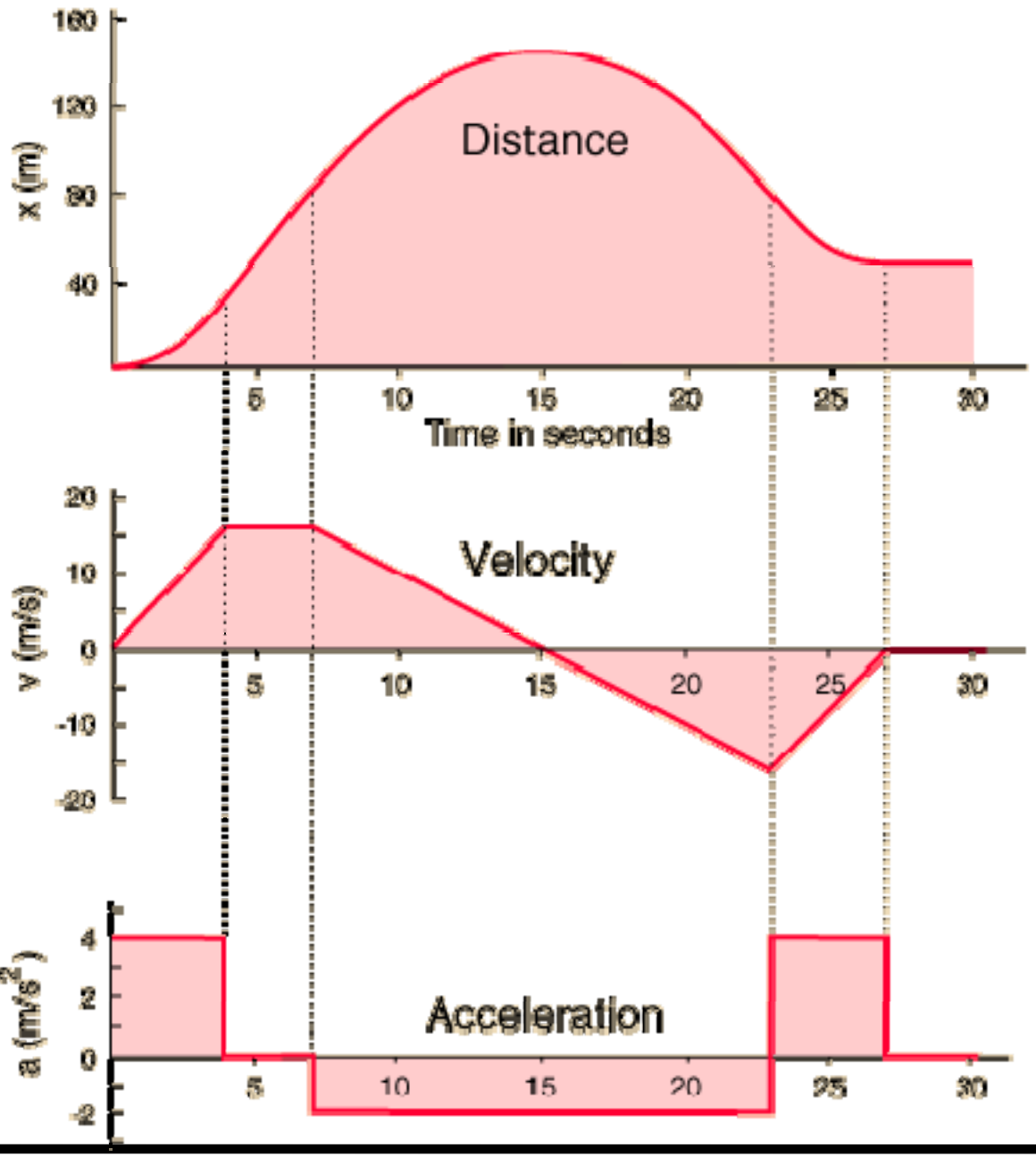
The Fundamental Theorem of Calculus, Part 1

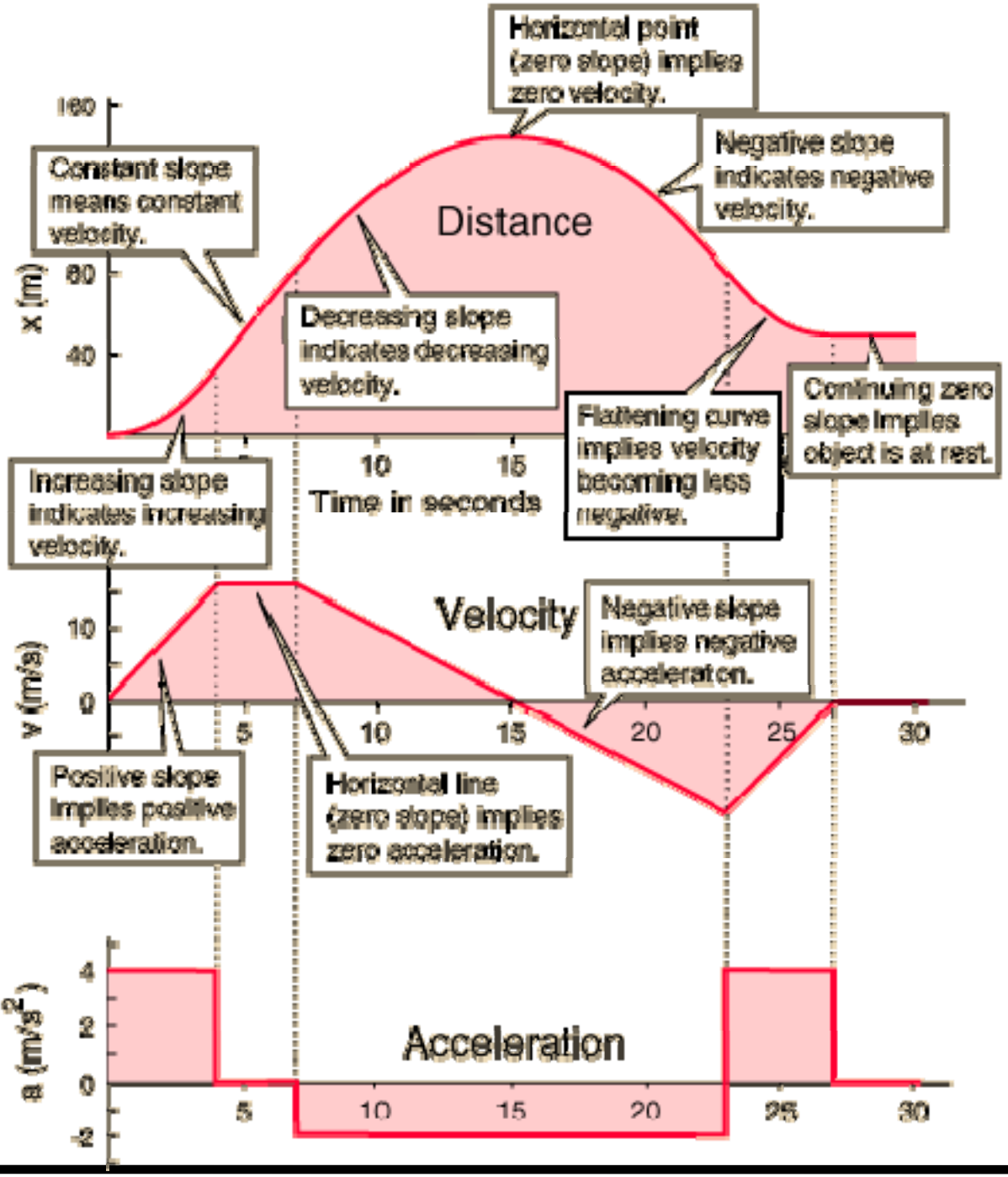
If f is continuous on $[a, b]$, then function g defined by

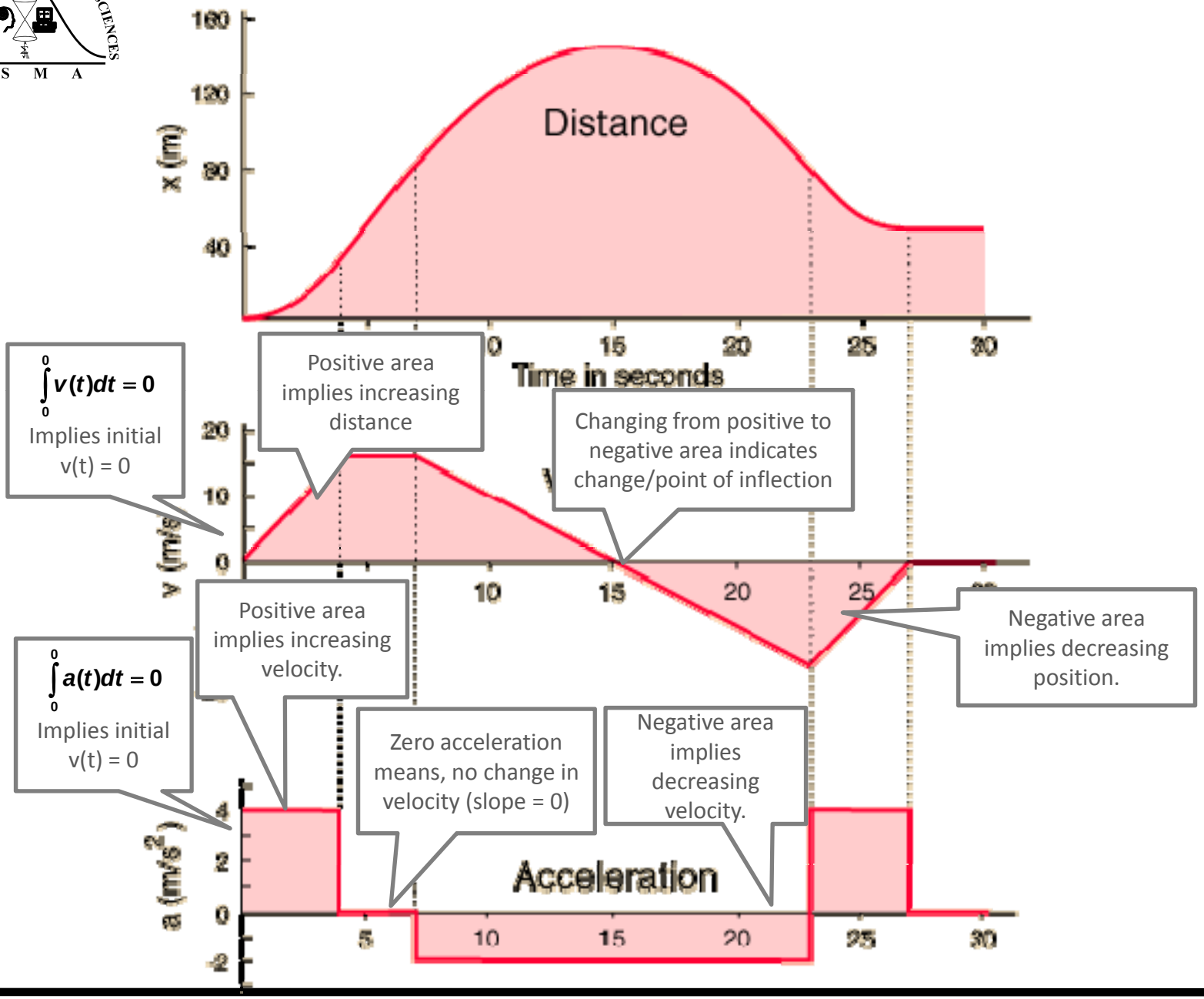
$$g(x) = \int_a^x f(t)\ dt \qquad a \le x \le b$$

is an antiderivative of f, that is, $g'(x) = f(x)$ for $a < x < b$.

Stewart, p. 381


Distance
x (m)
160
120
80
40
5
10
15
20
25
30
Time in seconds
Velocity
v (m/s)
20
10
0
-10
-20
Acceleration
a (m/s²)
4
2
0
-2


Horizontal point (zero slope) implies zero velocity.
Negative slope indicates negative velocity.
Constant slope means constant velocity.
Distance
Decreasing slope indicates decreasing velocity.
Flattening curve implies velocity becoming less negative.
Continuing zero slope implies object is at rest.
Increasing slope indicates increasing velocity.
Time in seconds
Velocity
Negative slope implies negative acceleration.
Positive slope implies positive acceleration.
Horizontal line (zero slope) implies zero acceleration.
Acceleration


Distance
x (m)
Time in seconds
$\int_0^0 v(t)dt = 0$
Implies initial v(t) = 0
Positive area implies increasing distance
Changing from positive to negative area indicates change/point of inflection
Positive area implies increasing velocity.
Negative area implies decreasing position.
$\int_0^0 a(t)dt = 0$
Implies initial v(t) = 0
Zero acceleration means, no change in velocity (slope = 0)
Negative area implies decreasing velocity.
Acceleration
a (m/s²)

Definition

The Fundamental Theorem of Calculus, Part 2

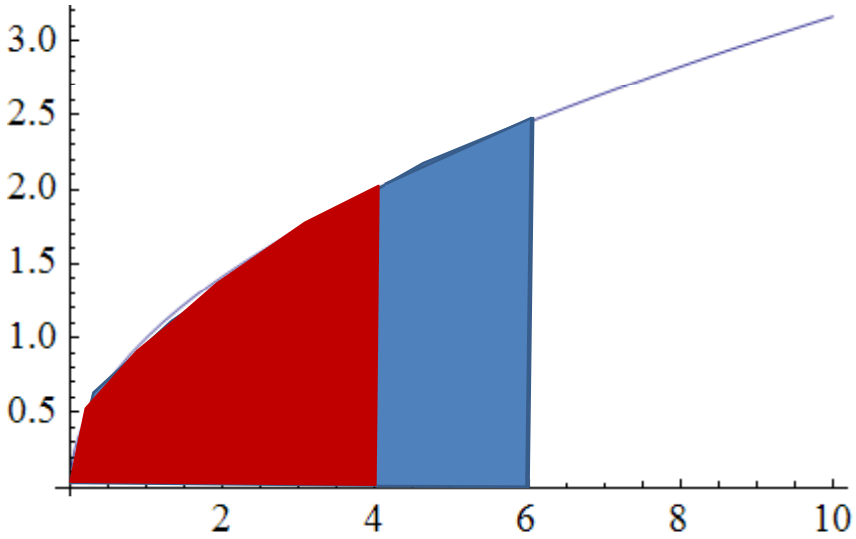
If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\ dx = F(b) - F(a)$$

Where F is an antiderivative of f, that is a function such that $F' = f$.

Stewart, p. 384

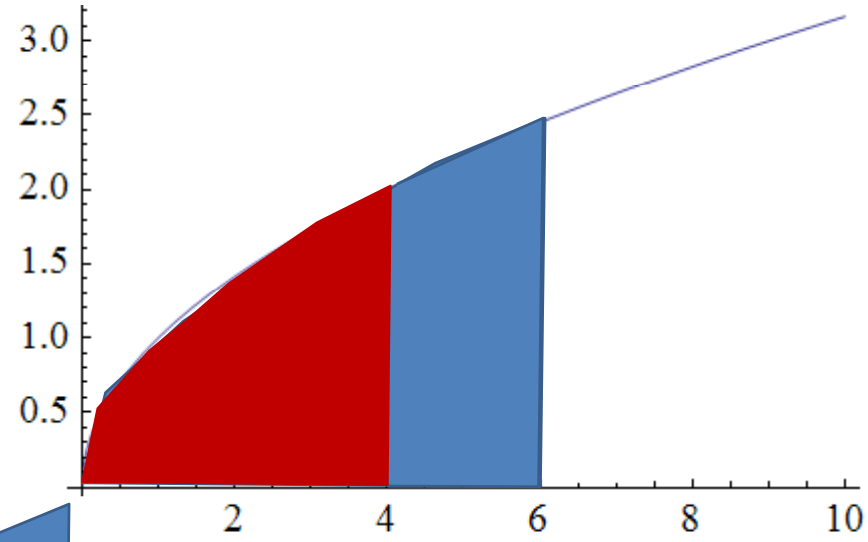
Graphically

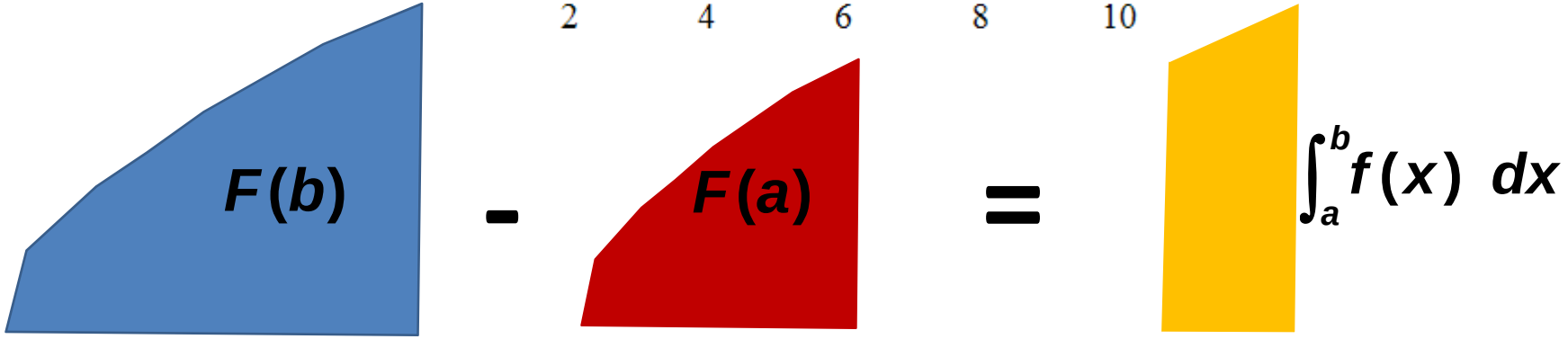


$$\int_a^b f(x)\ dx = F(b) - F(a)$$

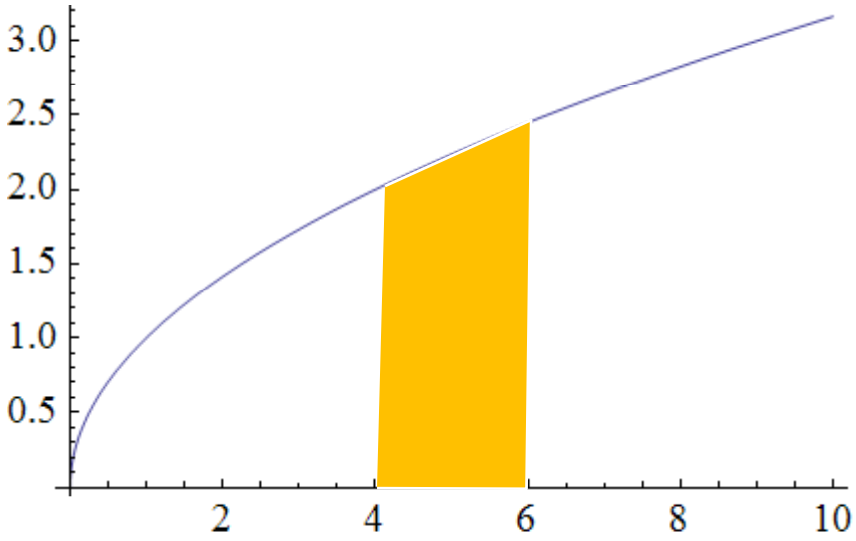
$$\int_4^6 f(x)\ dx = F(6) - F(4)$$

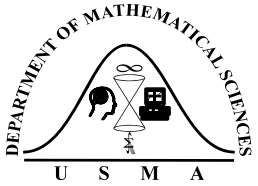
Graphically





Graphically





Back Up Slides

http://appsci.queensu.ca/courses/APSC171/mathQ/mathqs/ftc.html

Further References

http://blog.wolfram.com/2008/01/19/mathematica-and-the-fundamental-theorem-of-calculus/

http://math.fullerton.edu/mathews/n2003/Web/Calculus2Mod/Calculus2Mod.html