Lesson 3

Estimates From Functions and Data Sets II

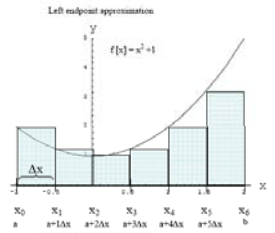
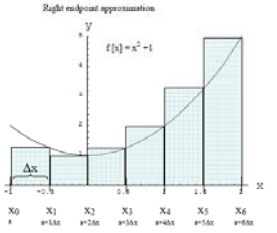
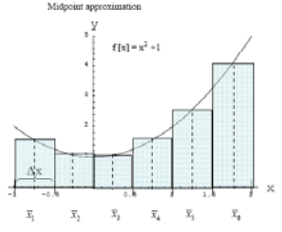
I must have assistants who will solve their own problems and tell me later what they have done.

General Marshall to General Eisenhower

Objectives

- **Choose sample points that lead to upper and lower estimates.**
- **Estimate the area of a general region using left end points and right end points.**
- **Estimate total distance traveled.**
- **Estimate the area between a curve and the x axis numerically when given a function using left end points, right end points, midpoints and trapezoids.**

Review

Approximation Method	Formula	Mathematica Code	Plot
Left Endpoint	$L_n = \sum_{i=1}^{n} f(x_{i-1})\Delta x$ $\Delta x = \frac{b-a}{n}$	$\Delta x = \frac{b-a}{n}$; x[i_] = a + i Δx; $L[n\_] = \sum_{i=1}^{n} f[x[i-1]]\,\Delta x$	
Right Endpoint	$R_n = \sum_{i=1}^{n} f(x_i)\Delta x$ $\Delta x = \frac{b-a}{n}$	$\Delta x = \frac{b-a}{n}$; x[i_] = a + i Δx; $R[n\_] = \sum_{i=1}^{n} f[x[i]]\,\Delta x$	
Midpoint	$M_n = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$ $\Delta x = \frac{b-a}{n}$, $\bar{x}_i = \frac{(x_{i-1}+x_i)}{2}$	$\Delta x = \frac{b-a}{n}$; x[i_] = a + i Δx; $M[n\_] = \sum_{i=1}^{n} f\left[\frac{x[i-1]+x[i]}{2}\right]\Delta x$	
Trapezoidal	$T_n = \frac{1}{2}\sum_{i=1}^{n}[f(x_{i-1})\Delta x + f(x_i)\Delta x]$ $\Delta x = \frac{b-a}{n}$	$\Delta x = \frac{b-a}{n}$; x[i_] = a + i Δx; $T[n\_] = \sum_{i=1}^{n}\left[\frac{f[x[i-1]]+f[x[i]]}{2}\right]\Delta x$