MA205 D2 AY2008-01 MAJ DeGregory, Keith
Lesson 51 16 NOV 07

Name: Answer Key (10 minutes)

References: You may use Stewart text, a reference sheet and a calculator.

Quiz 10

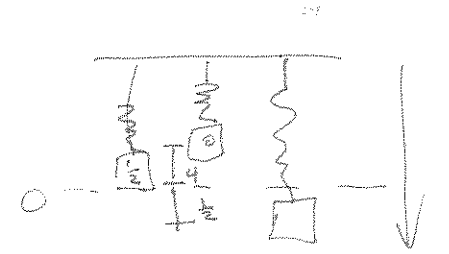
A 16 pound weight stretches a spring 4 ft from its natural position. The spring-mass system is in a medium with a damping constant of 4.5 lb-sec/ft.

1. **(8 points)** What is the particular solution describing the position of the mass at any time if the mass is released from 6 inches below the equilibrium position with an initial velocity of 4 ft/sec upward? Include a diagram.

$k = 4$ $\quad 16 = k(4)$ $\quad f = ma$

$c = 4.5$ $\quad k = 4$ $\quad 16 = m(32)$

$m = \frac{1}{2}$ $\quad m = \frac{1}{2}$



$\frac{1}{2}x'' + 4.5x' + 4x = 0, \quad x(0) = \frac{1}{2}, \quad x'(0) = -4$

$\frac{1}{2}r^2 + 4.5r + 4 = 0$

$r = \frac{-4.5 \pm \sqrt{4.5^2 - 4(\frac{1}{2})(4)}}{2(\frac{1}{2})} = -4.5 \pm \sqrt{20.25 - 8}$

$= -4.5 \pm 3.5$

$r_1 = -8 \quad r_2 = -1$

$\boxed{x(t) = \frac{1}{2}e^{-8t} \quad \text{ANS}}$

$x(t) = C_1 e^{-8t} + C_2 e^{-t}$ $\quad x(0) = \frac{1}{2} = C_1 + C_2$ $\quad C_1 = \frac{1}{2} - C_2$

$x'(t) = -8C_1 e^{-8t} - C_2 e^{-t}$ $\quad x'(0) = -4 = -8C_1 - C_2$ $\quad -4 = -8(\frac{1}{2} - C_2) - C_2$

$-4 = -4 + 8C_2 - C_2$

$0 = 7C_2 \quad C_2 = 0$

$C_1 = \frac{1}{2}$

Bonus (**2 Point**) Find the minimal spring constant that would produce under-damped behavior.

$c^2 - 4mk < 0$

$4.5^2 - 4(\frac{1}{2})k < 0$

$20.25 < 2k$

$k > 10.125$