Name: Solutions

AY 2009-1 MA205 Section 36 Hour: ALL

Lesson 50, 12 Nov 2008

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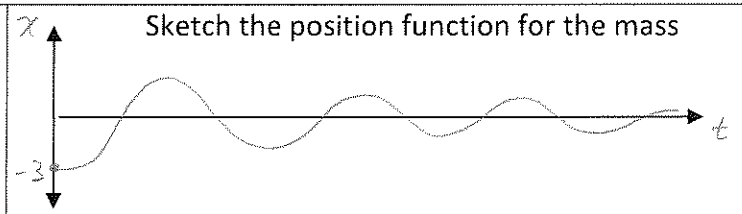
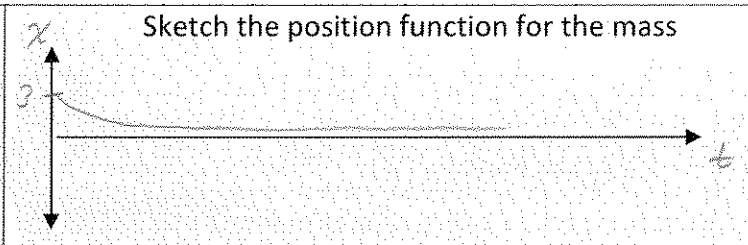
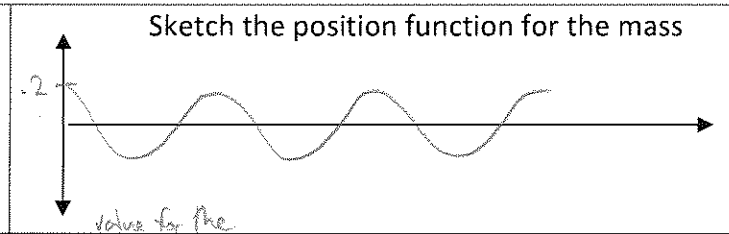
Quiz 8: 2nd Order Differential Equations and Spring Mass Systems

A few lessons back we had three spring mass systems on the front desk. One had no damping and the other two were damped (one system was in water and the other system was Pete Dawkins' head). Below is a chart of the model and solutions to three similar systems. For each system: classify the specific type of damping, draw a labeled sketch of the solution on the axis provided, and answer the corresponding question.

1) Model: $20x'' + 8x' + 28x = 0$ $x(0) = -3, x'(0) = 10$	Solution: $x(t) = e^{-0.2t}(-3\cos(1.17t) + 8.06\sin(1.17t))$	Sketch the position function for the mass 
Type of System: underdamping	Describe the long term behavior of this system. The mass will oscillate about equilibrium but at a decreasing amplitude.	
2) Model: $10x'' + 80x' + 150x = 0$ $x(0) = 3, x'(0) = 0$	Solution: $x(t) = 7.5e^{-3t} - 4.5e^{-5t}$	Sketch the position function for the mass 
Type of System: overdamping	What is maximum distance from equilibrium that the mass achieves and when does this occur? $X_{max} = 3$ occurs at $t = 0$.	
3) Model: $5x'' + 98x = 0$ $x(0) = .2, x'(0) = 0$	Solution: $x(t) = .2Cos(4.43t)$	Sketch the position function for the mass 
Type of System: un damped (Simple harmonic motion)	If we were to add resistance to this system, what value for the damping constant would be required to produce critical damping? $c^2 - 4mk = 0$ $c^2 = 4(5)(98)$ $c = \pm\sqrt{1960}$ since c is a positive constant $c = \sqrt{1960}$ Ans OR 44.27 Can we achieve this in real world? no, only in theory	