AY2009-1 MA205 Section 36
Lesson 21

MAJ DeGregory, Keith
25 SEP 08

Name: Solution

Hour:_____

## Quiz 4: Parametric Equations and Vectors

| Time: 15 Minutes | References: Basic Calculator, Ref. Sheet, Text book, and computer (no connectivity) | Based out of 100% |
|---|---|---|

1. Find the position vector function $\vec{r}(t)$ of a Patriot missile given its acceleration vector function $\vec{a}(t) = 7t\,\hat{i} - \frac{t^3}{2}\hat{j} + \sin(t)\hat{k}$, its initial position and velocity vector $\vec{v}(0) = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{r}(0) = 3\hat{i} - 1\hat{j} + 1\hat{k}$, respectively.

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \int \left(7t^2 - \frac{t^3}{2}\hat{j} + \sin(t)\hat{k}\right) dt$$

$$= \left(\frac{7}{2}t^2 + C_1\right)\hat{i} + \left(-\frac{t^4}{2\cdot 4} + C_2\right)\hat{j} + \left(-\cos(t) + C_3\right)\hat{k}$$

side order of equations

① $\frac{7}{2}(0)^2 + C_1 = 1 \Rightarrow C_1 = 1$

② $-\frac{0^4}{8} + C_2 = 2 \Rightarrow C_2 = 2$

③ $-\cos(0) + C_3 = -1 \Rightarrow C_3 = 0$

$$\vec{r}(t) = \int \vec{v}(t)\,dt = \int \left(\left(\frac{7}{2}t^2 + 1\right)\hat{i} + \left(-\frac{t^4}{8} + 2\right)\hat{j} + \left(-\cos(t)\right)\hat{k}\right) dt$$

$$= \left(\frac{7}{2\cdot 3}t^3 + 1t + D_1\right)\hat{i} + \left(\frac{-t^5}{8\cdot 5} + 2t + D_2\right)\hat{j} + \left(-\sin(t) + D_3\right)\hat{k}$$

① $\frac{7}{6}0^3 + 1(0) + D_1 = 3 \Rightarrow D_1 = 3$

② $\frac{-0^5}{40} + 2(0) + D_2 = -1 \Rightarrow D_2 = -1$

③ $-\sin(0) + D_3 = 1 \Rightarrow D_3 = 1$

$$\vec{r}(t) = \left(\frac{7}{6}t^3 + t + 3\right)\hat{i} + \left(-\frac{t^5}{40} + 2t - 1\right)\hat{j} + \left(-\sin(t) + 1\right)\hat{k}$$ ANS

2. Describe the position vector found above as an equivalent set of parametric equations.

$$x(t) = \frac{7}{6}t^3 + t + 3$$

$$y(t) = -\frac{t^5}{40} + 2t - 1$$

$$z(t) = -\sin(t) + 1$$

ANS.

3. What is the missile's coordinate point in space after 3 seconds.

$$\vec{r}(3) = \left(\frac{7}{6}(3)^3 + 3 + 3\right)\hat{i} + \left(\frac{-3^5}{40} + 2(3) - 1\right)\hat{j} + \left(-\sin(3) + 1\right)\hat{k}$$

$$= 37.5\,\hat{i} - 1.075\,\hat{j} + 0.859\,\hat{k}$$

$$(x, y, z) = (37.5, -1.075, 0.859)$$

ANS.