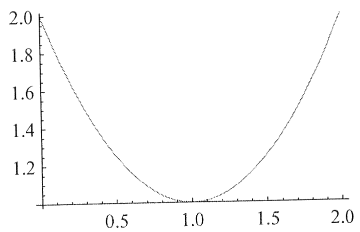
MA205 - Integral Calculus
Lesson 5: Problem Solving Lab I

1. The velocity of a particle over the time interval 0 to 2 seconds is given by $f(t) = t^2 - 2t + 2$ (in meters per second). Find upper and lower bounds on the total distance traveled. Show that a more accurate method produces a value in between your upper and lower bounds.

```
In[1]:= f[t_] = t^2 - 2*t + 2;

In[2]:= Plot[f[t], {t, 0, 2}]
```

Out[2]=



Note that $f(t)$ is decreasing on $[0,1]$ and increasing on $[1,2]$.

So using the left endpoint method on $[0,1]$ and " " right " " " $[1,2]$ and adding will give us an overestimate (upper bound).

If I use $n=5$, total dist ≈ 2.88 meters

Using the right endpoint method on $[0,1]$ and the left endpoint method on $[1,2]$, and adding gives us an underestimate (lower bound) of

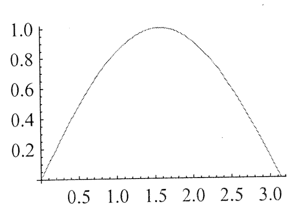
when $n=5$, total dist $= 2.48$ meters

With $n=5$ $\rightarrow$ midpoint method yields an estimate of 2.66
Trapezoid " " " " " 2.68
both within bounds

2. What is the *exact* area under the curve $f(x) = \sin(x)$ from $x = 0$ to $x = \pi$?

```
In[1]:= a = 0;
        b = π;
        f[x_] = Sin[x];

In[4]:= deltaX = (b - a)/n;
        x[i_] = a + i * deltaX;
        M[n_] = Sum[f[(x[i - 1] + x[i])/2], {i, 1, n}] * deltaX;

In[52]:= Plot[f[x], {x, a, b}]
```

Out[52]=



I don't know exactly, so I can estimate using the midpoint method which yields the following:

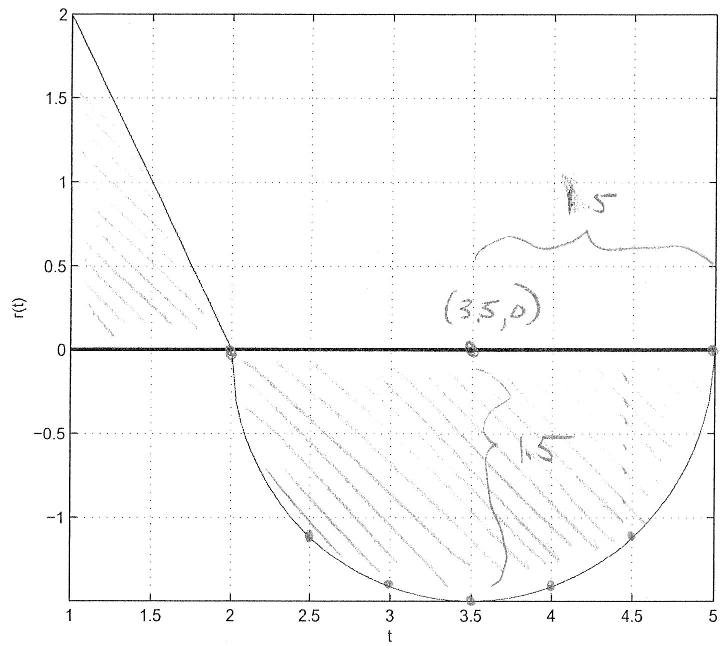
```
In[9]:= M[1] // N
Out[9]= 3.14159

In[10]:= M[10] // N
Out[10]= 2.00825

In[12]:= M[15] // N
Out[12]= 2.00366

In[13]:= M[100] // N
Out[13]= 2.00008

In[15]:= M[1000] // N
Out[15]= 2.

In[16]:= Limit[M[n], n → ∞]
Out[16]= 2
```

note as n gets big, $M(n)$ seems to converge to 2.

as $n \Rightarrow \infty$

$M(n) \rightarrow 2$ Ans

3. Below is a graph of the rate of change of oil in a container, which we denote by the function $r(t)$ (the units are in hundreds of gallons per minute).



(a) What is happening to the oil between times $t = 1$ and $t = 2$? How about from $t = 2$ to $t = 5$?

(b) Physically, what is the meaning of $\int_1^5 r(t)\, dt$?

(c) Graphically, what is the meaning of $\int_1^5 r(t)\, dt$? Sketch it on the graph above.

(d) Calculate $\int_1^5 r(t)\, dt$.

a.) The rate of change of oil in the container is decreasing; oil is being added into the container, but it is being added more slowly as t goes from 1 to 2.

From t = 2 to t = 5, the oil is being drained from the tank, at an increasing speed from 2 to 3.5 minutes, and decreasing speed from 3.5 to 5 minutes.

b.) $\int_1^5 r(t)\,dt$ is the total amount of oil (in gallons) that has been added to the tank from t = 1 to t = 5.

c) Shaded in the graph above

d) $\frac{1}{2}(2)(2) = 2$ (area from t = 1 to t = 2)

$\frac{1}{2}(\pi(1.5)^2) = \frac{9\pi}{8}$ (from t = 2 to t = 5 is $\frac{1}{2}$ a circle of radius $\frac{3}{2}$)

Total = $\boxed{2 + \frac{9\pi}{8}}$ = Ans