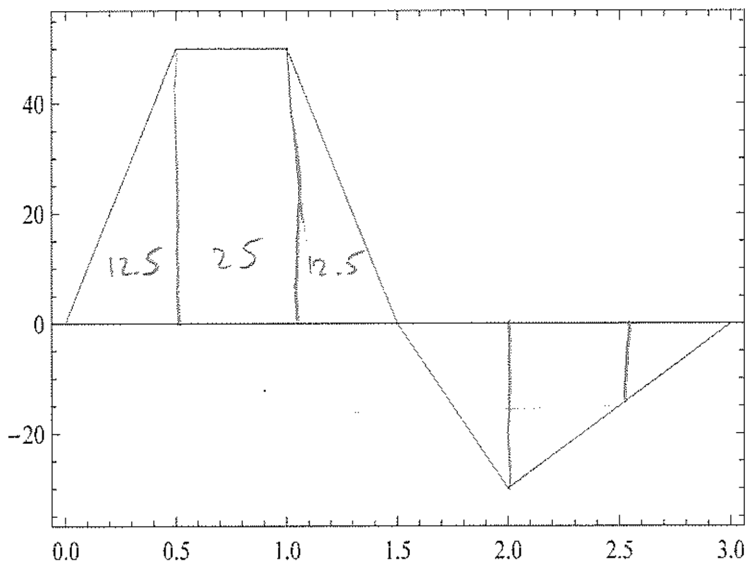
MA205 - Integral Calculus
Lesson 7: The Fundamental Theorem of Calculus I

1. Recall a problem from last lesson: Below is a graph representing the velocity of a car, given by the function $v(t)$ (in meters per hour), versus time (in hours). Find the following quantities. What do they mean to the driver?



We found that $\int_0^{1.5} v(t)\,dt = 50$, $\int_{1.5}^{3} v(t)\,dt = -22.5$, and therefore $\int_0^{3} v(t)\,dt = 27.5$.

Now, find the following values:

(a) $\int_0^{0} v(t)\,dt = 0$ meters

(b) $\int_0^{.5} v(t)\,dt = \frac{1}{2}(50)(.5) = 12.5$ meters

(c) $\int_0^{1} v(t)\,dt = \int_0^{.5} v(t)\,dt + \int_{.5}^{1} v(t)\,dt = 12.5 + \frac{1}{2}(50) = 37.5$ meters

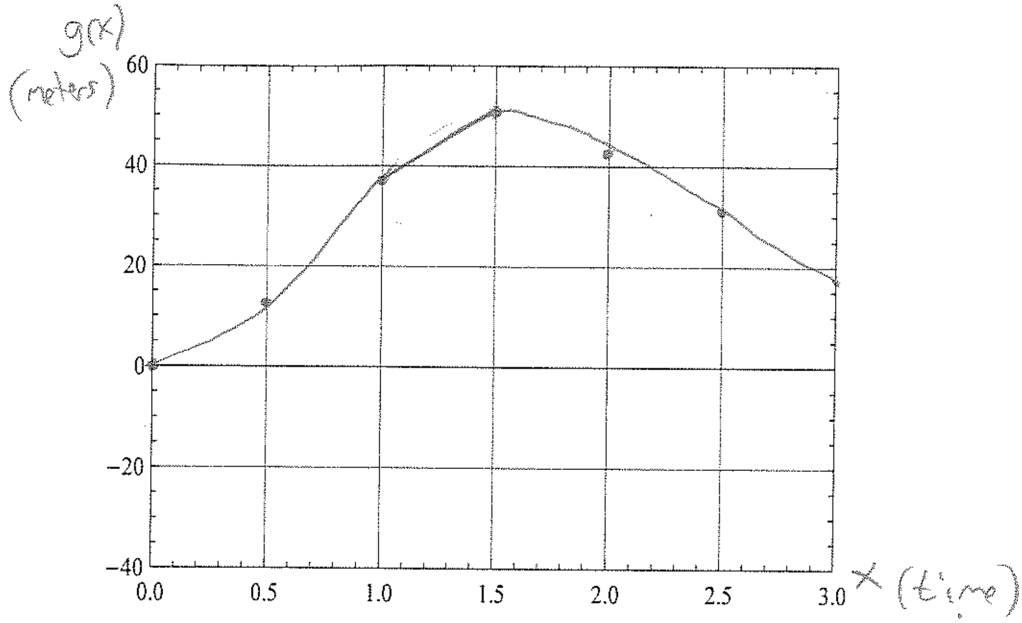
(d) $\int_0^{1.5} v(t)\,dt = 37.5 + \int_1^{1.5} v(t)\,dt = 37.5 + \frac{1}{2}(50)(.5) = 50$ meters

(e) $\int_0^{2} v(t)\,dt = 50 + \int_{1.5}^{2} v(t)\,dt = 50 - \frac{1}{2}(30)(.5) = 42.5$ meters

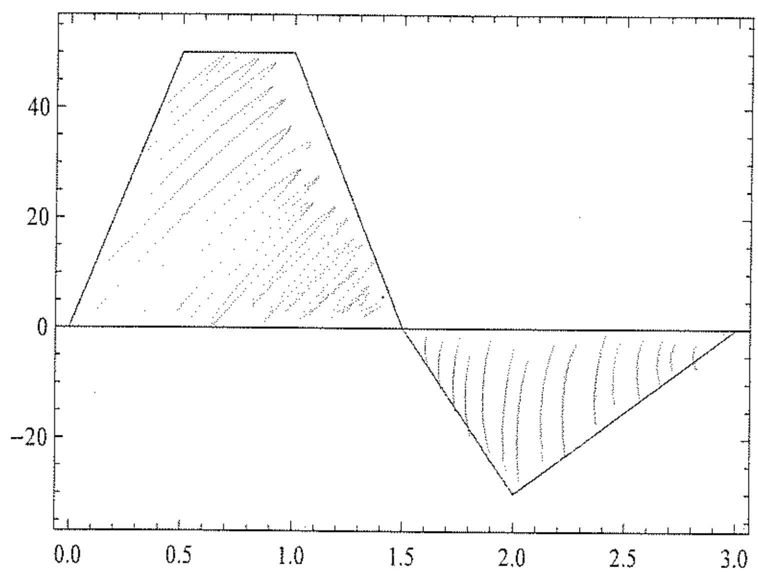
(f) $\int_0^{2.5} v(t)\,dt = 42.5 + \int_2^{2.5} v(t)\,dt = 42.5 - \frac{1}{2}(30+15)(.5) = 31.25$ meters

(g) $\int_0^{3} v(t)\,dt = 31.25 + \int_{2.5}^{3} v(t)\,dt = 31.25 - \frac{1}{2}(15)(.5) = 27.5$ meters

If we defined the values we just calculated with a function: $g(x) = \int_0^x v(t)\,dt$, then plot these values on the chart below. Label your axes with the correct units.



and compare this NEW function with our original function, $v(t)$.



What does this new plot represent?

This new plot represents the area under the original function as the value x changes.