MA205 - Integral Calculus
Lesson 8: The Fundamental Theorem of Calculus II
Board Problems

Find the most general antiderivative of the function:

1. $f(x) = 3\sin x$ $\quad F(x) = 3(-\cos x) + C = \underline{-3\cos x + C}$ Ans

2. $f(x) = 6x + 12x^2$ $\quad F(x) = \frac{6x^2}{2} + \frac{12x^3}{3} + C = \underline{3x^2 + 4x^3 + C}$ Ans

3. $f(t) = 2e^t + 3\sin t$ $\quad \underline{F(t) = 2e^t - 3\cos t + C}$ ANS

4. $g(x) = \frac{2}{x} + 6\cos x$ $\quad \underline{G(x) = 2\ln|x| + 6\sin x + C}$ Ans

5. $h(\theta) = \left(\frac{100}{\theta} + 50\right)$ $\quad \underline{H(\theta) = 100\ln|\theta| + 50\theta + C}$ Ans

6. $f(y) = 10\sqrt[3]{y} + 15\cos(y) + \sin(t)$ $\quad y^{1/3}$ $\quad \underline{F(y) = \frac{10y^{4/3}}{4/3} + 15\sin y + \sin(t)\cdot y + C}$ ANS

7. $f(x) = x^{-1/3}$ $\quad F(x) = \frac{x^{-1/3+3/3}}{2/3} + C = \underline{\frac{3}{2}x^{2/3} + C}$ ANS

Compute the following definite integrals, using the Fundamental Theorem of Calculus, Part 2.

1. $\int_1^2 3\sin(x)\,dx = -3\cos x\Big]_1^2 = \underline{-3\cos(2) - (-3\cos(1))}$

2. $\int_{-1}^3 6x + 12x^2\,dx = 3x^2 + 4x^3\Big]_{-1}^3 = \underline{\left[3(3)^2 + 4(3)^3\right] - \left[3(-1)^2 + 4(-1)^3\right]}$ Ans

3. $\int_0^1 2e^t + 3\sin t\,dt = 2e^t - 3\cos t\Big]_0^1 = \underline{\left[2e^1 - 3\cos(1)\right] - \left[2e^0 - 3\cos 0\right]}$ ANS

4. $\int_{1}^{2} \frac{2}{x} + 6\cos x\,dx = 2\ln|x| + 6\sin x\Big]_0^1 = \underline{\left[2\ln|2| + 6\sin(2)\right] - \left[2\ln|1| + 6\sin(1)\right]}$

5. $\int_5^{10} \left(\frac{100}{\theta} + 50\right)\,d\theta = 100\ln|\theta| + 50\theta\Big]_5^{10} = \underline{\left[100\ln 10 + 50(10)\right] - \left[100\ln 5 + 50(5)\right]}$

6. $\int_2^{1000} 10\sqrt[3]{y} + 15\cos(y) + \sin(t)\,dy = \frac{30}{4}y^{4/3} + 15y + \sin t\cdot y\Big]_2^{1000}$

$= \left[\frac{30}{4}(1000)^{4/3} + 15(1000) + \sin t\,(1000)\right]$

$\underline{- \left[\frac{30}{4}(2)^{4/3} + 15(2) + \sin t\,(2)\right]}$ Ans

7. $\int_{-.5}^{.5} x^{-1/3}\,dx$

$= \frac{3}{2}x^{2/3}\Big]_{-.5}^{.5}$

$= \left(\frac{3}{2}(.5)^{2/3}\right) - \left(\frac{3}{2}(-.5)^{2/3}\right)$ // Ans

Find f.

1. $f'(x) = 2x - \frac{3}{x^4}$, $f(-1) = 0$

$$\int f'(x)\,dx = f(x) = \int 2x - 3x^{-4}\,dx$$

$$= \frac{2x^2}{2} - \frac{3x^{-3}}{-3} + c = x^2 + x^{-3} + c = f(x)$$

If $f(-1) = 0$, then

$(-1)^2 + (-1)^{-3} + c = 0$

$1 + (-1) + c = 0$

$c = 0$

$\rightarrow$ So $\underline{f(x) = x^2 + x^{-3}}$ ANS

2. $f'(x) = 2 - 12x + e^x$, $f(0) = 9$

$$f(x) = \int 2 - 12x + e^x\,dx = 2x - \frac{12x^2}{2} + e^x + c = f(x)$$

If $f(0) = 9$, then

$2(0) - 6(0)^2 + e^0 + c = 9$

$1 + c = 9$

$c = 8$

So $\rightarrow$ $\underline{f(x) = 2x - 6x^2 + e^x + 8}$ ANS

3. $f'(x) = 2\cos x + x^{\frac{1}{3}}$, $f(\pi) = 3$

$$f(x) = \int 2\cos x + x^{1/3}\,dx$$

$$= 2\sin x + \frac{3}{4}x^{4/3} + c$$

If $f(\pi) = 3$, then

$2(\sin(\pi)) + \frac{3}{4}(\pi)^{4/3} + c = 3$

$0 + \frac{3}{4}\pi^{4/3} + c = 3$

$c = 3 - \frac{3}{4}\pi^{4/3}$

$\rightarrow$ So $f(x) = 2\sin x + \frac{3}{4}x^{4/3} + \left(3 - \frac{3}{4}\pi^{4/3}\right)$ ANS