MA205 - Integral Calculus
Lesson 9: Indefinite Integrals and Net Change

1. Given the function, find the indefinite or definite integral. Check your work with Mathematica.

(a) $\int -4e^t \, dt$ $= -4\int e^t dt = -4(e^t + c) = -4e^t + 4C$

this is still arbitrary

$\Downarrow$

$\underline{\underline{-4e^t + C}}$ ANS

(b) $\int (-3\sin(x) + 1) \, dx$ $= -3\int \sin(x)dx + \int 1 dx$

$= -3(-\cos(x) + c) + (x + c)$

$= 3\cos(x) - 3c + x + c \Rightarrow \underline{\underline{3\cos(x) + x + c}}$ ANS

(c) $\int_3^4 \frac{1+x}{x^2} \, dx$ $= \int_3^4 \frac{1}{x^2}dx + \int_3^4 \frac{x}{x^2}dx = \int_3^4 x^{-2}dx + \int_3^4 \frac{1}{x}dx$

$= -\frac{1}{x}\Big]_3^4 + \ln|x|\Big]_3^4$

$= -\frac{1}{4} - \left(-\frac{1}{3}\right) + \ln|4| - \ln|3| \underline{\underline{\approx 0.37}}$ ANS

(d) $\int_1^9 (1 + \sqrt{p})^2 \, dp$ $= \int_1^9 (1 + 2\sqrt{p} + p)dp$

$= \int_1^9 1 dp + 2\int_1^9 p^{\frac{1}{2}} dp + \int_1^9 p dp$

$= p\Big]_1^9 + 2\left(\frac{p^{3/2}}{3/2}\right)\Big]_1^9 + \frac{p^2}{2}\Big]_1^9$

$= [9 - 1] + \frac{4}{3}[9^{3/2} - 1^{3/2}] + \frac{1}{2}[9^2 - 1^2] = \underline{\underline{82.7}}$ ANS.

(e) $\int \frac{47}{u} + \frac{1}{u^2} \, du$ $= 47\int \frac{1}{u}du + \int u^{-2}du$

$= 47(\ln|u| + c) + \frac{u^{-1}}{-1} + C$

$= \underline{\underline{47\ln|u| - \frac{1}{u} + c}}$ ANS

t^3+4t+t^2+4

(f) $\int_{-\sqrt{t}}^{\sqrt{t}} (t+1)(t^2+4)\, dt = \int_{-\sqrt{t}}^{\sqrt{t}} t^3+t^2+4t+4\, dt$

$$= \frac{t^4}{4} + \frac{t^3}{3} + \frac{4t^2}{2} + 4t \Big]_{-\sqrt{t}}^{\sqrt{t}}$$

$$= \frac{t^{4/2}}{4} + \frac{t^{3/2}}{3} + 2t^{2/2} + 4t^{1/2} - \left(\frac{(-t^{\frac{1}{2}})^4}{4} + \frac{(-t^{\frac{1}{2}})^3}{3} + 2(-t^{\frac{1}{2}})^2 + 4(-t^{\frac{1}{2}})\right)$$

$$= \cancel{\frac{t^2}{4} - \frac{t^2}{4}}^{\,0} + \frac{t^{3/2}}{3} + \frac{t^{3/2}}{3} + \cancel{2t - 2t}^{\,0} + 4\sqrt{t} + 4\sqrt{t} = \underline{\underline{\frac{2}{3}t^{3/2} + 8\sqrt{t}}}$$ ANS

2. The velocity function (in meters per second) is given as $v(t) = t^2 - 2t - 8$ for a particle moving along a line over a 3-second interval, starting with $t = 0$. Find

(a) the displacement of the particle during the given time interval.

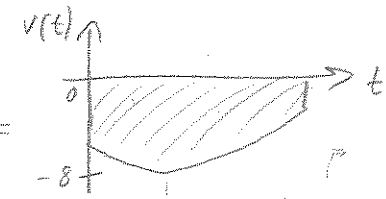
$$s(t) = \int_0^3 v(t)\, dt = \frac{t^3}{3} - t^2 - 8t \Big]_0^3 = \frac{3^3}{3} - 3^2 - 8(3) - [\,0\,]$$

$$= \underline{\underline{-24 \text{ meters}}}$$ ANS

(b) the distance traveled by the particle during the given time interval.

$x(t)$ = distance traveled at time t

$$x(t) = \int_0^3 |v(t)|\, dt = |-24| - |0| = \underline{\underline{24 \text{ meters}}}$$ ANS



3. The acceleration function (in m/s^2) is given as $a(t) = 2t + 3 - 20\sin(t)$ for a particle moving along a line over a 10-second interval, starting with $t = 0$, where $v(0) = -4\ m/s$. Find

(a) the velocity at time t of the particle during the given time interval.

$$v(t) = \int_0^{10} a(t)\, dt = t^2 + 3t + 20\cos(t) + C$$

$$v(0) = -4 = 0^2 + 3(0) + 20\cos(0) + C$$

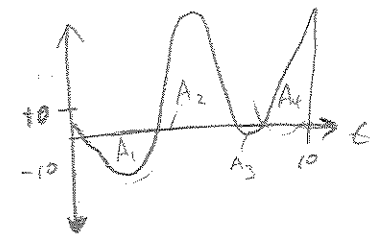
$$-4 = 20 + C$$

$$C = -24 \implies \underline{\underline{v(t) = t^2 + 3t + 20\cos(t) - 24 \text{ meters/sec}}}$$ ANS

(b) the distance traveled by the particle during the given time interval.

Mrn Norm [v[t]]

$$x(t) = \int_0^{10} |v(t)|\, dt = \underline{\underline{232.45 \text{ meters}}}$$ ANS



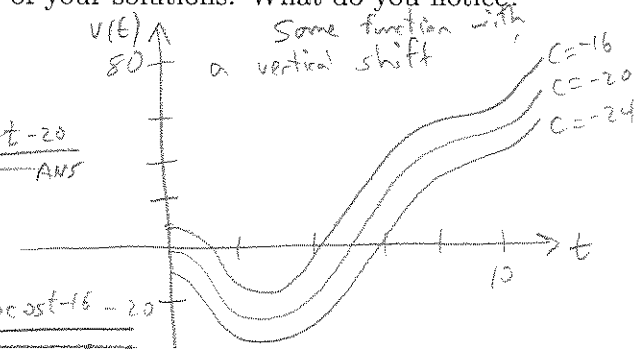
4. See the previous problem. Answer question (a), assuming the same acceleration function, but where $v(0) = 0\ m/s$. What if $v(0) = 4\ m/s$? Plot each of your solutions. What do you notice?

$$v(0) = 0 = 0^2 + 3(0) + 20\cos(0) + C$$

$$0 = 20 + C$$

$$C = -20 \qquad \underline{\underline{v(t) = t^2 + 3t + 20\cos t - 20}}$$ ANS

$$v(0) = 4 \implies 4 = 20 + C$$

$$C = -16$$

$$\underline{v(t) = t^2 + 3t + 20\cos t - 16 - 20}$$