AY2009-1 MA205 Section 36
Lesson 13

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10 SEP 08

Name: Solution

Hour:_____

Quiz 2: Integrals and FTC

Time: 15 Minutes	References: Basic Calculator	Based out of 100%
For full credit either explain and fully justify your reasoning or clearly demonstrate all work using proper mathematical notation.		

1. If $g(x)=\int_2^x e^t\,dt$ demonstrate mathematically that $g'(x)=e^x$.

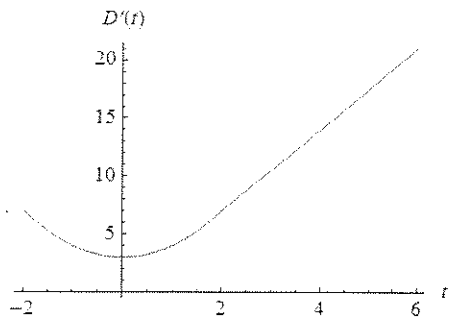
$g(x)=\int_2^x e^t\,dt = e^t\Big]_2^x = e^x - e^2$

$g(x)=e^x-e^2$

$g'(x)=e^x$

ANS.

2. Given the piecewise function $D'(t)=\begin{cases} t^2+3 \text{ for } t\le 2 \\ 3.5t \text{ for } t>2\end{cases}$ find $\int_{-2}^{6} D'(t)dt$.



$\int_{-2}^{6} D'(t)\,dt = \int_{-2}^{2} t^2+3\,dt + \int_2^6 3.5t\,dt$

$= \frac{1}{3}t^3+3t\Big]_{-2}^{2} + \frac{3.5}{2}t^2\Big]_2^6$

$= \left(\frac{1}{3}(2^3)+3(2)-\left[\frac{1}{3}(-2)^3+3(-2)\right]\right)+\left(1.75(6)^2-1.75(2^2)\right)$

$= \frac{8}{3}+6-\left(-\frac{8}{3}-6\right)+1.75(36-4) = \frac{52}{3}+56 = 17.\overline{3}+56 = 73.\overline{3}$

ANS

3. A vehicle traveling on a straight road accelerates according to $a(t)=4t+6$ feet/sec^2. How much has the vehicle displaced between 10 and 20 seconds given the velocity at time five was 50 feet/sec.

$V(t)=\int 4t+6\,dt$

$= 2t^2+6t+C$

$V(5)=2t^2+6t+C=50$

$50+30+C=50$

$C=-30$

$V(t)=2t^2+6t-30$

$\int_{10}^{20} 2t^2+6t-30\,dt$

$= \frac{2}{3}t^3+3t^2-30t\Big]_{10}^{20}$

$= \frac{2}{3}(20)^3+3(20)^2-30(20)-\left(\frac{2}{3}(10)^3+3(10)^2-30(10)\right)$

$= \frac{16000}{3}+1200-600-\left(\frac{2000}{3}+3(100)-300\right) \approx 5333.3-666.7 \approx 5266.7$ feet

ANS

4. Relate the Riemann Sum (approximation using the sum of rectangles) to the definite integral.

$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)\,dx$ OR $\sum_{i=1}^{n} f(x_i^*)\Delta x \approx \int_a^b f(x)\,dx$

ANS