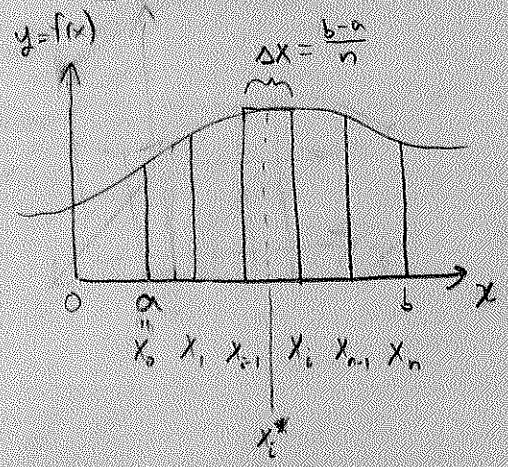
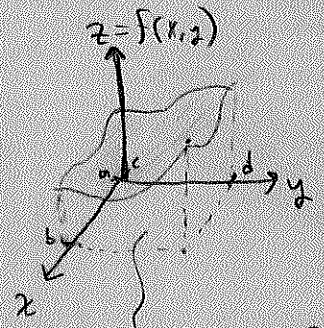
(1) Review of Def. Integral



$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x$$

"Area under the curve"

(2) Functions of Sev. Variables

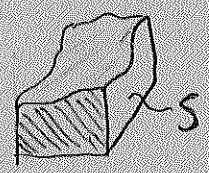


$R=[a,b]\times[c,d]$

$V = h\times l\times w$ $\quad$ x = length, y = width, z = height

$= z\,\Delta x\,\Delta y$

$= f(x,y)\,\Delta A$

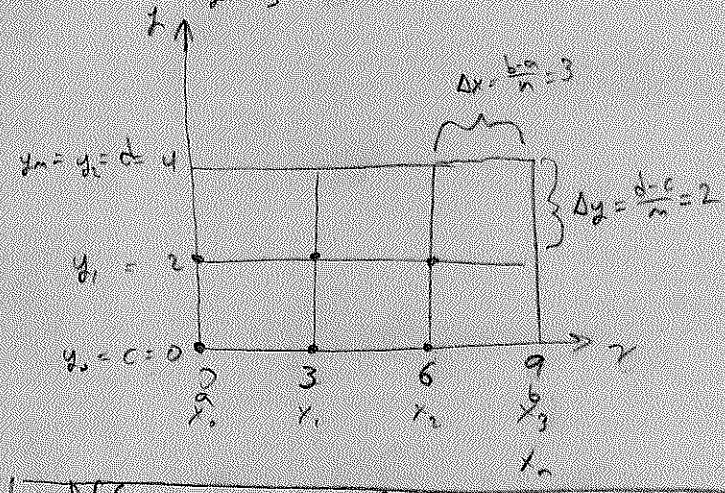


(6) 2D Plot of Region

Domain

$D=\{(x,y)\mid 0\le x\le 9,\ 0\le y\le 4\}$

6 rectangles $\Rightarrow$ 6 heights



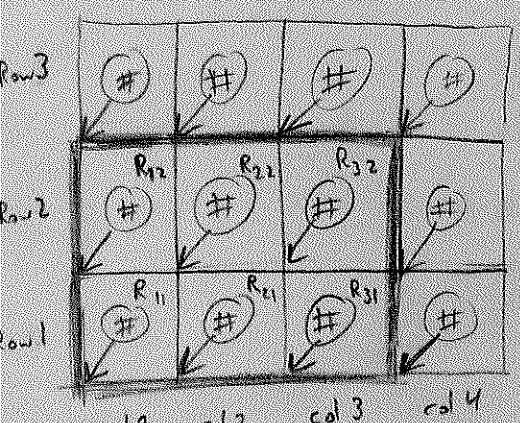
(7) Lof 6

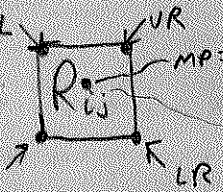
$$V=\iint_R f(x,y)\,dA = \lim_{m,n\to\infty}\sum_{j=1}^{m}\sum_{i=1}^{n} f(x_i,y_i)\Delta A = \int_c^d\int_a^b f(x,y)\,dx\,dy$$

(3) Data Table (3D)

x	z
0	0
1	1
2	4

} 2D





col $\to i$

row $\to j$

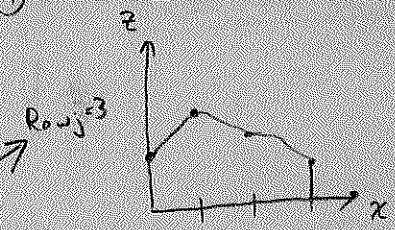
$MP = \left(\frac{UL+UR+LL+LR}{4}\right)$

$f(\bar{x}_i,\bar{y}_j)$

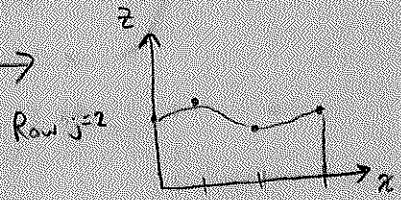
(4) 2D Cross-section of Slices

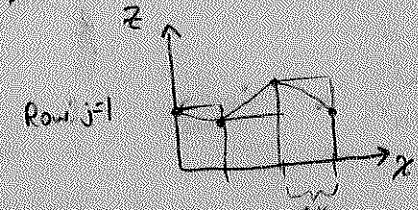
Range

$\{z \mid z=f(x,y),\ (x,y)\in D\}$



$A_j = \sum_{i=1}^{n} f(x_i,y_j)\Delta x$





$A_1 = \sum_{i=1}^{3} f(x_i,y_1)\Delta x$

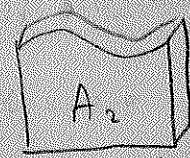
$= [f(x_1,y_1)+f(x_2,y_1)+f(x_3,y_1)]\Delta x$

$(\#+\#+\#)3 =$

(5)

$V_j = \sum_{i=1}^{n} f(x_i,y_j)\Delta x\,\Delta y$

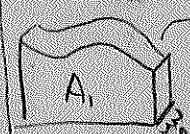
+



V_2

$$V_5 = \sum_{j=1}^{m-2} V_j = (V_1+V_2) = \sum_{j=1}^{2}\sum_{i=1}^{3} f(x_i,y_j)\Delta x\,\Delta y$$

+



$V_1 = \sum_{i=1}^{3} f(x_i,y_1)\Delta x\,\Delta y$

$= (\#)(3)(2)$