MA205 – Integral Calculus and Introduction to Differential Equations
Project III – 150 Points

The following assignment is due **NLT 29 NOV 07 in class**. In accordance with the Documentation of Written Work (DWW), you must document any and all collaboration with sources other than your partner, your textbook, or your instructor. Documentation must be IAW Paragraph 1 (Documentation Standard), Section VI (Complying With Documentation Standards) of the DWW. Work will be assessed for your group performance. Evaluation and grading will account for the use of sources in addition to your text book. Sufficient work is required to indicate clearly the method of reasoning and the operations performed. **SHOW ALL WORK.**

Background

Lake Euler, the "Distribution Center" lake your team has analyzed in your previous two reports to the board of Stark Enterprises, is actually one in a series of three lakes that are interconnected:

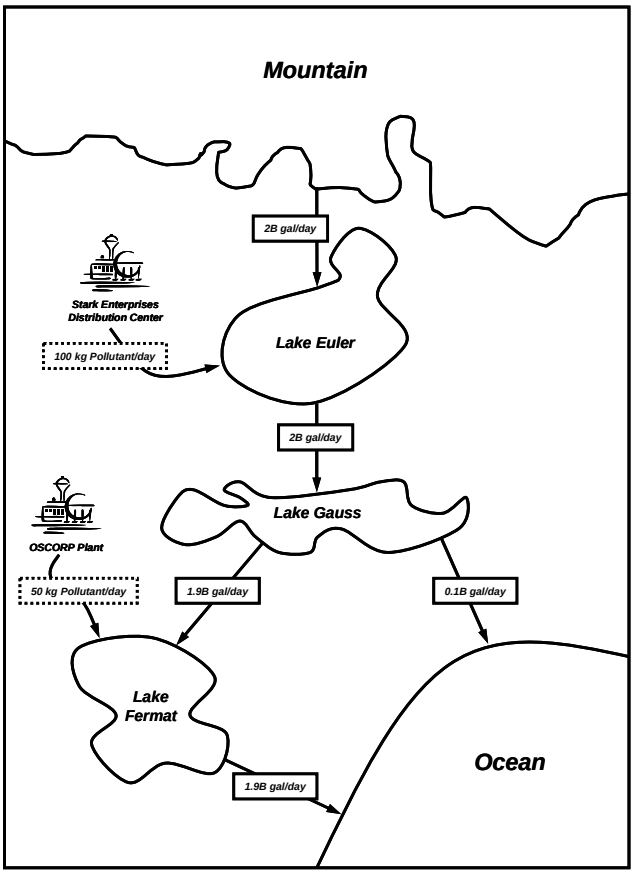

Figure 1. Interconnected Tri-Lake Waterway System

Stark Enterprises was recently contacted by the Environmental Protection Agency (EPA) regarding elevated arsenic levels in this tri-lake waterway system. EPA records show that an OSCORP Plant on the nearby Lake Fermat maintained sufficiently low arsenic levels for the five years prior to the construction of the Stark Enterprises Distribution Center (SEDC). Within the past year that the SEDC has been in operation, the arsenic levels in all three lakes in the waterway system (Lake Euler, Lake Fermat, and Lake Gauss) have escalated well above EPA standards. This indicates, at least to the EPA, that all current arsenic contamination issues are directly attributable to Stark Enterprises.

Option A

The EPA has given Stark Enterprises six months to bring the waterway system (all three lakes as well as the interconnecting streams) in compliance or the SEDC will be shut down. The Stark Enterprises staff has gathered the following data from their company records and existing EPA policies:

- The tri-lake waterway system is composed of Lake Euler, Lake Gauss, and Lake Fermat, as shown in Figure 1 (Interconnected Tri-Lake Waterway System). The volumes of each lake are estimated at 191 billion gallons, 150 billion gallons, and 175 billion gallons, respectively. Lake Euler is fed fresh water from a mountain river at a rate of 2 billion gallons per day, and outputs this same flow daily into Lake Gauss. Lake Gauss has a small tributary that feeds directly into the ocean at a rate of 0.1 billion gallons per day, and a larger river flows to Lake Fermat at a rate of 1.9 billion gallons per day. Lake Fermat's only tributary feeds directly into the ocean, also at a rate of 1.9 billion gallons per day.

- The EPA standards state that marine environment arsenic contamination levels may not exceed 10 parts per billion. An example of how this standard is applied is that a lake having a ***mass*** of 1 billion kg of water cannot have more than 10 kg of arsenic contamination. The OSCORP Plant is dumping 50 kg of arsenic per day into Lake Fermat and the SEDC is dumping 100 kg of arsenic per day into Lake Euler. It is estimated that there is currently 9341 kg of arsenic in Lake Euler, 6946 kg in Lake Gauss, and 11479 kg in Lake Fermat.

- The EPA stipulates that Stark Enterprises' arsenic remediation plans not require OSCORP to incur any costs and that Stark Enterprises is fully responsible for any disposal (as required) of OSCORP arsenic wastes throughout. It will cost Stark Enterprises $2.50 to transport 25 kg of arsenic from the OSCORP Plant to the SEDC. Stark Enterprises' only alternative to dumping arsenic into Lake Euler is to collect and dispose off-site some (or all) of the 100 kg produced per day (as well as whatever arsenic is collected from the OSCORP Plant), at a cost of $3 per kg of arsenic.

- The EPA will conduct a monitoring stage of Stark Enterprises' remediation plan after the 180 day cleanup period.

The Board of Directors for Stark Enterprises requests your team's services in developing a two-phase arsenic remediation plan:

1. Determine the cost and time associated with bringing arsenic levels within compliance for the tri-lake waterway system, as achieved by immediately suspending all arsenic dumping into Lake Euler and Lake Fermat.

2. Once the tri-lake waterway system is within compliance, provide a recommendation and cost analysis for the long-term arsenic management of both Lake Euler and Lake Fermat. Specifically, the CEO would like to know when arsenic dumping can be resumed for both the SEDC and OSCORP Plant, in what amounts, and at what cost.

Provide a report to the CEO with your analysis, and make any recommendations that he should implement for the future. As usual, the CEO is interested in both your methodology and final results. Be sure to show all supporting work, state all assumptions, and give an overall recommendation of how the arsenic contamination problem should be handled.

Option B

Arsenic exists in water in two toxic forms: arsenite and arsenate. Arsenate is comparatively easy and safe to dispose of, but arsenite is not.

The CEO of Stark Enterprises has a personal interest in heavy metals and biotechnology, and he recently read an article that described some bioengineering work being done by researchers at La Trobe University in Australia. These researchers use bacteria to remove arsenic from contaminated wastewater on mining sites and from groundwater in Bangladesh and West Bengal, India. The research group is studying 13 rare bacteria that were isolated from gold mines in Australia. Theoretically, it is cheaper and safer to use bacteria to remove arsenic than existing chemical methods. One bacterium in particular, NT-26, was found to be able to eat arsenite and excrete arsenate.[1] The research group has found the enzyme directly responsible for converting arsenite to arsenate, which in turn will allow NT-26 to be grown on a mass scale. A key consideration is to find the stable balance between the amount of arsenic and the amount of NT-26 in a water supply, as the correct balance leads to a self-sustaining system. As such, Stark Enterprises would like to investigate a bioremediation strategy for the arsenic contamination problem in the tri-lake waterway system.

The interaction between NT-26 and water-borne arsenic is modeled by the following system of ***nonlinear*** differential equations:

$$\frac{du_{Arsenic}}{dt} = u_{Arsenic}(1 - u_{Arsenic} - \gamma\ u_{NT-26})$$
$$\frac{du_{NT-26}}{dt} = u_{NT-26}(1 - u_{NT-26} - \delta\ u_{Arsenic})$$

Equation 1. Mutualism Model Describing Interaction Between Water-Borne Arsenic and Bacterium NT-26

$u_{Arsenic}$ is a variable describing the proportion of an aggregate arsenic amount to the tri-lake waterway system's carrying capacity for arsenic. Similarly, u_{NT-26} represents the proportion of total biomass of NT-26 to the tri-lake waterway's carrying capacity for that bacterium. Note that since these two variables describe a proportion, they are dimensionless. The two parameters in the system, γ and δ, also dimensionless, $0 < \gamma, \delta < 1$, are known as ***control parameters*** – they represent the effects on the arsenic amount and bacterium population that result from continuous removal of the arsenate that NT-26 excretes, and from the fact that NT-26 has to be continuously introduced into the waterway system (since some of the bacterium population is washed away into the ocean daily).

The advantage of a bioremediation approach is that it is self-adaptive: if the NT-26 population in the waterway system and arsenic amounts stabilize to some co-existent level, it does not matter if Stark Enterprises continues to dump arsenic into Lake Euler, even at current amounts. The catch is that NT-26 cannot rid the waterway system of all the arsenic, just some of it. And Stark Enterprises still has to pay for removal/disposal of the resultant arsenate through some other means. However, as mentioned previously, the cost (and safety) of arsenate removal is much preferred over the cost of arsenite removal.

[1] www.irc.nl, "Arsenic Removal: Research on Bioremediation Using Arsenite-Eating Bacteria"

The EPA standards state that marine environment arsenic contamination levels may not exceed 10 parts per billion. This translates into a value of $u_{Arsenic} = 0.7143$. Also, the NT-26 population has to be managed in such a way that $0.287 \le u_{NT-26} \le 0.659$, else the NT-26 population will die off from other environmental factors.

Since bioremediation is an emerging field, it is expensive. Specifically, management of the NT-26 population and resultant arsenate waste removal will cost Stark Enterprises $3 million annually for every unit of u_{NT-26}. However, these expenses are offset by a governmental tax credit as computed by $125,000 times the ratio $0.7143/u_{Arsenic}$.

The CEO of Stark Enterprises requests your team's services in determining both the feasibility and the cost of this bioremediation strategy. Specifically, determine values (if they exist) for the control parameters γ and δ in the NT-26/arsenic interaction model such that:

1. **The long-term amount of arsenic and long-term biomass of NT-26 in the tri-lake waterway system are stable.**
2. **The long-term amount of arsenic in the tri-lake waterway system is below EPA minimum standards and the cost impact to Stark Enterprises is reasonably low, i.e. less than $1.5 million annually.**

Assume that if a feasible bioremediation strategy is found, then it will be implementable and sustainable within the six month suspense set forth by the EPA. Provide a report to the CEO with your detailed analysis. Your report must include your determination of the feasibility of the bioremediation strategy, your recommended target levels for both NT-26 and arsenic for the tri-lake system, and the overall cost of implementing the strategy. As usual, the CEO is interested in both your methodology and final results. Be sure to show all supporting work, state all assumptions, and give an overall recommendation of how the arsenic contamination problem should be handled.