WPR III Topics

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1. Basics of Differential Equations

   - What is a differential equation, and what does it mean to be a solution to one?
   - Test if you have a solution to a differential equation (including for IVPs)
   - Vocabulary: order, linearity, and homogeneity of a DE; dependent and independent variables; stable and unstable equilibrium solutions.
   - Take a sentence or paragraph and turn it into a differential equation or IVP.

2. Methods for Solving Differential Equations

   - Graphical: Slope fields
     - Benefits and drawbacks
     - Use a slope field to sketch solution curves (by hand)
     - Use a slope field and solution curve to answer other questions about the behavior of the solution.
   - Numerical: Euler's Method
     - Understand how Euler's method works.
     - Benefits and drawbacks
     - Execute Euler's method by hand for a limited number of steps.
     - Execute Euler's method with a small step size, using technology.
   - Analytic: Separation of variables (First order)
     - Execute separation of variables *by hand* to solve a first order DE. (This includes finding a particular solution to an IVP by using an initial condition.)
     - Note: You have to know algebra and how to integrate by hand to do these problems!!!
   - Analytic: Second order
     - Solve a second order, linear, homogeneous DE *by hand*.
     - Find the characteristic equation.
     - Use the roots of the characteristic equation to determine the form of the general solution.
     - Use initial conditions (and algebra!!) to solve for both constants c_1 and c_2 and write down a particular solution.
     - Note: You have to know how to take derivatives (chain rule, product rule, etc) and the quadratic formula to do these problems!!!

3. Applications of Differential Equations

   - Exponential growth and decay
     - Know the form of an exponential growth/decay DE and the form of the solution.
     - Know how to solve for the constant k in one of these problems.
     - Newton's Law of Cooling: What is it? How do you solve these?

  - Population growth, radioactive decay, continuously compounded interest: these are all exponential examples!
- Logistic model
  - Why is the logistic model a more realistic population model than an exponential one?
  - What is a "carrying capacity"?
  - What are the equilibrium solutions of a logistic equation? Classify them as stable and unstable (this includes recognizing them on a slope field).
  - Which solutions have inflection points?
  - What is the long term behavior?
- Electric circuits
  - Vocabulary!!! Capacitor, resistor, inductor, current, charge, electromotive force: know the mathematical variable, the units, and the diagram symbol (where appropriate).
  - Know the voltage drops over resistors, inductors, and capacitors.
  - What is the relationship between current and charge?
  - Turn a diagram of an electric circuit into a DE by using Kirchoff's second law. (We had 2 models for this.)
- Spring Mass
  - What is the differential equation that models the position of a spring?
  - Compute the spring constant, mass, and damping constant given various types of information.
  - Solve these by hand or using Mathematica.
  - Understand how to convert given information about the spring into initial conditions for the DE.
  - Classify damping as overdamped, critically damped, or underdamped and understand the meaning of each.
  - How does the model change if a driving force is introduced?

4. Mathematica Excitement :)

- Plot a slope field, together with a solution curve.
- Euler's method
- Execute the `DSolve` command and utilize its result to answer further questions.