MA205 - Integral Calculus
Lesson 25: General Regions I

To integrate over a general region:

1. Neatly draw the label the region. (Solve for intersection points if necessary.)
2. Describe the region in terms of inequalities. Ask yourself:
   - What happens as y increases (from the bottom to the top of the region)?
   - What happens as x increases (from the left to the right of the region)?

   Note: Describe either x *or* y (not both!!) in terms of functions and the other in terms of extreme values.
3. Use (2) to choose an order of integration and set up the integral. The extremes of your inequalities will be your limits of integration. *Be sure* that the outside limits of integration are *numbers*!
4. Integrate, starting on the *inside* and working your way *outside*.

Problems

1. Integrate the following functions over the given region. Make sure to draw and label the region.

   (a) $f(x, y) = x^2 + y$, D is the region bounded by the curves $y = 1 - x$, $x = 0$ and $y = x - 1$.

   (b) $f(x, y) = x^2 y$, D is the region bounded by the curves $y = -x^2 + 1$, $x = 0$ and $y = x - 1$.

(c) $f(x, y) = x + y$, D is the the region bounded by the line $x = 4$ and the parabola $y^2 = x$.

2. Sketch the region of integration and change the order of integration:

$$\int_0^1 \int_{4x}^4 f(x, y)\, dydx$$

3. Evaluate $\int_0^1 \int_{3y}^3 e^{x^2}\, dxdy$ by reversing the order of integration.