MA205 - Integral Calculus
Lesson 26: General Regions II

Reminders:

- One variable (x or y) will be described in terms of functions and the other one will be described in terms of extreme values.
- The outer limits of integration must be *numbers*, not functions!
- An accurate graph (including labels) is *essential* to working these problems!

Problems

1. Here's an integral where we *must* switch the order of integration. Consider

$$\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy.$$

(a) First, draw a careful picture of the region that is dictated to you by the double integral. Be sure to include labels.

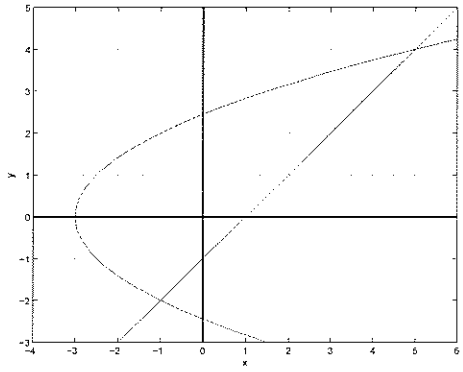
Note: In the original integral, x is described in terms of functions and y is described in terms of numbers (extreme values). We need to reverse this.

(b) What are the extreme values of x in this region? Which integral (inside out outside) will this translate to?

(c) Draw a vertical line in your region and use it to determine the starting and ending points of y. (These should be functions now!)

(d) Now set up the double integral and calculate the answer.

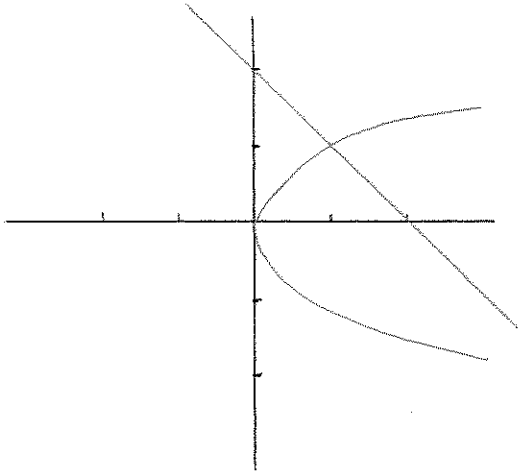
2. Set up an integral that will calculate $\int\int_D f(x,y)\,dA$, where D is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$. The order you choose matters here; choose wisely!



3. Sometimes you can take two integrals and combine them into one. Consider the following:

$$\int_1^2 \int_0^{2-y} xy\,dx\,dy + \int_0^1 \int_0^{y^2} xy\,dx\,dy.$$

Switch the order of integration and combine the integrals into one. Then calculate the integral. (How do you tackle this problem? Draw the region for each integral on the same set of axes. Then try to use one double integral to describe the common region.)



4. Consider the region D shaded below. Set up the double integral $\int\int_D f(x,y)\,dA$ in two different ways: once as a $dydx$ integral and once as a $dxdy$ integral.