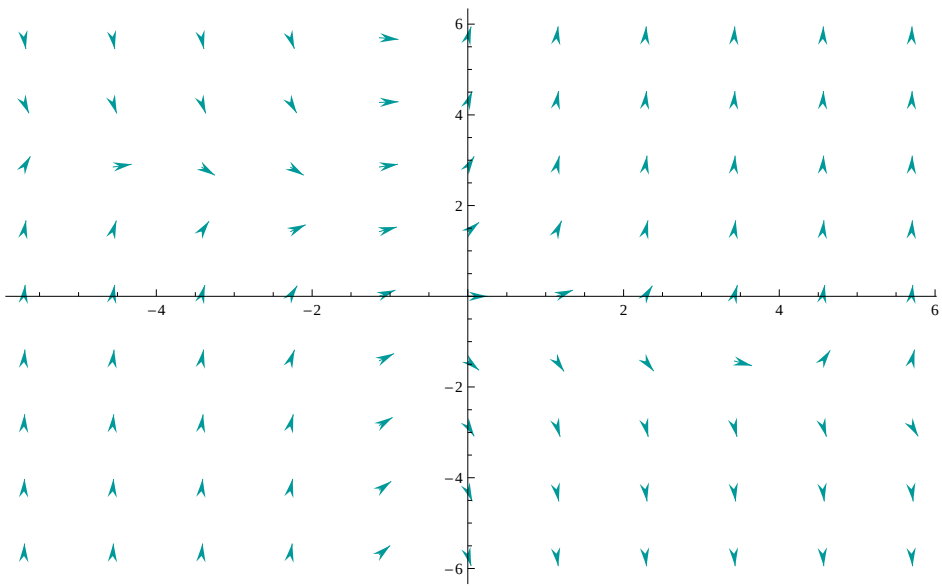
MA205 - Integral Calculus
Lesson 42/43: Slope Fields and Euler's Method

1. On the slope field given below, sketch the solution which satisfies:

   (a) $y(0) = 2$

   (b) $y(2) = 3$



2. Now, using the Mathematica command we just learned, plot a direction field for the differential equation $y' = xy + \frac{x^2}{2} + y$. First, plot it with no initial conditions. Then, plot it with the initial conditions $y(0) = 2$ and $y(2) = 3$. These should look similar to your sketches in Problem 1.

3. Let's return to our ladybug problem from Tuesday. (See Problem 4 on the Lesson 41 handout.) We set up the differential equation

$$\frac{dP}{dt} = kP,$$

   where $P(t)$ is the population of ladybugs at any time t. Suppose that in addition, we know there are 200 ladybugs initially and 500 ladybugs after 3 days.

   (a) Use the two points given above to estimate the value of k.

   (b) Now that you have an estimate for k, plot a slope field for this differential equation, together with a solution curve. According to your graph, when does the population of ladybugs reach 1000? What is the long term behavior?

4. Use Euler's Method with a step size of 0.1 to estimate $y(1.4)$ where $y(x)$ is the solution of the initial value problem $y' = x - xy$, $y(1) = 0$.

   (a) Complete one iteration of Euler's method by hand. In other words, we are given (x_0, y_0), find (x_1, y_1) by hand.

   (b) Now use Mathematica to finish off the computation. Write down the table of x and y values it gives you, and use that to write down an estimate for $y(1.4)$.

5. A tank contains 1000L of brine with 20kg of dissolved salt. Brine that contains 2kg of salt per liter enters the tank at a rate of 5 L/min. The solution is kept well mixed and drains from the tank at the same rate.

   (a) Set up a differential equation that models the above system. Remember to name your variables! (Hint: Rate in - Rate out)

   (b) Use Euler's method (and an appropriate step size) to determine how much salt is left in the tank after 30 minutes.