MA383 Foundations in Mathematics
Lesson 4: Logic - Class Problems

1. Write the contrapositive and converse of each of the following conditional statements. Assume x is a real number throughout.

   (a) If $x + 1 > 5$ then $x > 4$.

   (b) If $x^2 - x - 2 = 0$ then $x = -1$ or $x = 2$.

   (c) If Sarah lives in Athens, then she lives in Greece.

2. Using DeMorgan's Laws, together with what we learned about the negation of an implication, write the negation of each of the following statements.

   (a) August is a hot month and September is sometimes cool.

   (b) If it is Monday, then I am tired.

   (c) If n is a prime number, then n is odd or n is 2.

3. Use truth tables to prove the following logical equivalencies.

   (a) $\neg(\neg P \to Q) \equiv \neg P \wedge \neg Q$

Turn over for more logic fun!

(b) $(P \vee Q) \rightarrow R \equiv (P \rightarrow R) \wedge (Q \rightarrow R)$ [Note: Since you have to include all possibilities for truth values for P, Q, and R, you will need 8 rows in your table.]

4. Use previously proven logical equivalencies to prove the logical equivalence

$$(P \leftrightarrow Q) \equiv (\neg P \vee Q) \wedge (\neg Q \vee P).$$