

MA383 - Foundations in Mathematics
Homework Assignment 7
Due to TH233 by 1600 on Thursday, November 13, 2008

Remember, your writing and presentation will be graded in addition to the correctness of your solution!

1. Let $d : \mathbb{N} \rightarrow \mathbb{N}$, where $d(n)$ is the number of natural number divisors of n . (For example, $d(4) = 3$, since 4 has three natural number divisors, 1, 2, and 4.)

(a) Determine the set of preimages of 2 under d .

(b) Determine whether or not d is a bijection, and prove your conclusion. (Hint: Consider part (a).)

2. Define

$$f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \times \mathbb{R} \quad \text{by} \quad f(x, y) = (3x + y, -y - x).$$

(a) Determine whether or not f is injective, and prove your conclusion.

(b) Determine whether or not f is surjective, and prove your conclusion.

3. Define

$$p_1 : A \times B \rightarrow A \quad \text{by} \quad p_1(a, b) = a.$$

(This function is an example of a projection function.)

(a) Determine whether or not p_1 is a surjective function, and prove your conclusion.

(b) Suppose $B = \{b\}$. Is p_1 one-to-one? Prove your conclusion.

(c) Construct an example of sets A and B so that p_1 is not injective. Prove your result.

4. Complete problem 6a on page 316.