MA383 - Foundations in Mathematics
Lesson 34 - Review of Sets and Functions

Name: _____

Use this class period to work on the problems below. You may use your book, notes, old homeworks, and classmates as references. I will be happy to answer your questions, check your answers, or give hints if you get stuck. *If you would like the opportunity to win points to add to your WPR 2 score, please write out your own solutions and turn in them into me at the end of class today.* As always, please take care to show all your work and write neatly and clearly.

1. (2 points each) Mark each of the following as true or false. If a statement is true, tell me how you know it is true. If a statement is false, write a corrected statement in its place, or explain why it is false.

   _____ (a) If $f : A \to B$ and $B_1 \subseteq B$, then $f^{-1}(B_1) \subseteq A$.

   _____ (b) $\{1\} \in \{1, 4, 5\}$

   _____ (c) $\{2, \pi, 4.5\}$ and $\mathbb{N}$ are disjoint sets.

   _____ (d) $\emptyset \in \mathcal{P}(A)$ for all sets A.

   _____ (e) $(\pi^2, 3) \in \mathbb{N} \times \mathbb{R}$

   _____ (f) If $f : A \to B$, then $\text{Range}(f) = B$.

   _____ (g) If $A = \{2, 3, 12\}$, then $|A| = 2^3 = 8$.

   _____ (h) If $f : A \to B$ and $g : B \to C$, then $f \circ g : A \to C$.

2. Define $h : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ by $h(m, n) = 4m + 8n$. Why is it impossible for h to map *onto* $\mathbb{Z}$?

3. Suppose A, B, and C are nonempty sets. Prove that

$$(A - B) - C \subseteq A - (B - C).$$

(Note: A Venn diagram will not suffice.)

4. Suppose $f : A \to B$ is a surjective function and suppose $C \subseteq A$. Prove that

$$B - f(C) \subseteq f(A - C).$$

5. Suppose $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is defined by $f(x, y) = 6x + 3y$ and $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ is defined by $g(x, y) = (6x + 3y, 6x + 5y)$. One of these is injective and one is not. Determine which is which, and prove your answers.