MA104 - WPR IV Topics

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1. Multivariable Functions and Partial Derivatives

   - Understand the concept of a function of 2 or 3 variables.
   - Know how to find the domain of a function of more than one variable.
   - Know what a level curve is.
   - Understand what is required for the limit of a function of 2 variables to exist at a particular point.
   - Know how to find the limit of a function of 2 variables by using a table of values, a contour plot, a 3D plot, or a formula for the function.
   - Understand how to use a contour plot to gain basic information about a 2 variable function. (e.g. estimate function values, see where function is high or low, where it changes sharply or gradually)
   - Know how to use a table of values to gain basic information about a 2 variable function. (e.g. function values, where it is increasing or decreasing)
   - Know how to compute first and second order partial derivatives of given functions *by hand*. (This means you have to know all your basic derivatives, plus the chain rule, product rule, and quotient rule!)
   - Know how to estimate values of first order partial derivatives by only using a table of function values or a contour plot.
   - Understand the *idea* of what rate of change each partial derivative is measuring, including its geometric meaning.

2. Directional Derivatives and the Gradient

   - Know how to compute the gradient of a function *by hand* (including at a specific point).
   - Know how to compute the directional derivative of a function *by hand* (including at a specific point). (Remember, this includes knowing how to compute a unit vector and a dot product!)
   - Understand the geometric meaning of the gradient vector, and how to plot the gradient vector on a contour plot.
   - Understand precisely what rate of change a directional derivative is measuring.
   - Understand that the gradient vector gives the direction of maximum rate of change of a function, and know how to use that to find the actual *amount* of maximum rate of change of the function at a given point.
   - Know how to use a contour plot to estimate a directional derivative at a point.

3. Extrema of Functions of 2 Variables

   - Know what a critical point is for a function of 2 variables and how to find them by hand.
   - Understand the idea of local and absolute maximum and minimum of a function $f(x, y)$.

- Know how to classify critical points (as local max, local min, or saddle point) using the 2nd Derivative Test. (Remember, you do not need to memorize this test, it will be given to you on the WPR.)
- Know how to find the absolute max and absolute min of a function of 2 variables on a closed and bounded domain. (i.e. know how to find critical points and compare their values to the values along the boundary of the domain.)
- Be able to use a contour plot to make an educated guess as to where the max/min of a function $f(x, y)$ is located (including on a closed and bounded region).

4. Optimization

- Be able to realize when a word problem is an optimization problem, by looking for words such as "minimize, maximize, largest, smallest, least, most, cheapest, etc."
- Be able to use the information in the problem to write down and *objective function*. (This is the function that represents what you want to maximize or minimize.)
- 2 variable optimization, *without* Lagrange multipliers
  - Be able to use extra information in the problem (often called a *constraint*) to eliminate variables and write your objective function in terms of only *two* variables. If the information given implies a closed and bounded domain for your function, find that domain.
  - Maximize or minimize your objective function using the techniques mentioned in the above section. Remember, this includes proving that your answer actually is the type of extreme that the problem is seeking. **It is essential that you show your work, even if you have Mathematica do the calculations for you!**
- Optimization using Lagrange multipliers
  - Identity your constraint equation.
  - Be able to solve the optimization problem by solving the system of equations given by Lagrange's method. This could be by hand (an easy one) or in Mathematica. Be able to interpret your results.
  - Understand how to locate a max/min of an objective function on a constraint by using the contour plot. (What geometric property will you always notice?)
- Remember to check that your answer makes sense, and is actually the type of extreme value you are looking for. Also, remember to be sure you are answering all the questions you have been asked, and include units if appropriate!
- In addition, it may be helpful for you to know what types of optimization problems are well suited for Lagrange's method, and what types may not be.

5. Mathematica things to know

- `ContourPlot`
- `Plot3D`
- Compute partial derivatives in Mathematica, using the `D` command or the palette.
- Compute a gradient vector in Mathematica, including at a specific point.
- Compute the directional derivative at a point, using Mathematica to do the leg work.
- Find critical points in Mathematica, and test them using the 2nd derivative test.
- Execute the Lagrange multiplier method in Mathematica.