MA104 - Differential Calculus
Lesson 23: Problem Solving with Related Rates I

When solving related rates problems, break the problem into a series of steps to be sure you're covering all the bases:

1. Draw a picture or diagram of what the problem describes.
2. Identify all quantities that are *changing* in the scenario, and *label them* in your picture.
3. Determine an equation that relates the above quantites and write it down.
4. Using implicit differentiation (i.e. the chain rule) take $\frac{d}{dt}$ of both sides of the above equation. (Hint: remember all of your changing quantities are really functions of time t!)
5. *After* taking the derivative, substitute in all given information.
6. Solve for the unknown, and be sure to check that your final answer *makes sense* and has units.

Example 1: A 17 foot ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/sec, how fast will the top of the ladder be moving down the wall when it is 8 feet above ground?

Example 2: A stone is dropped into the lake, creating a circular ripple that travels outward at a speed of 60 cm/s. At what rate is the area within the circle increasing after 3 seconds?

Problems

1. A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 10 ft/s. When he passes under the balloon it is 30 ft above him. How fast is the distance between the boy and the balloon increasing 2 seconds later?

2. Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of 6 mi^2/hr . How fast is the radius of the spill increasing when the area is 9 mi^2?

3. A heap of rubbish in the shape of a cube is being compacted. Given that the volume decreases at a rate of 2 cubic meters per minute, find the rate of change of an edge of the cube when the volume is exactly 27 cubic meters. What is the rate of change at that instant of the surface area of the cube?

4. Air is being pumped into a spherical balloon so that its volume increases at a rate of 120 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

5. **Challenge Problem:** A water tank has the shape of an inverted circular cone with a top radius of $2m$ and height $4m$. If water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the radius of the surface of the water is increasing when the water is $3m$ deep.