MA104 - Differential Calculus
Lesson 29: Parametric Equations II

1. Two vehicles, a Seacar and an Oceancycle, have been designed to drive along the ocean floor. They have both been sent to examine the floor of a particular cave, and are driving along two different paths. Using the sonar in our pod, we plot the Seacar at position $(1,4)$ and the Oceancycle at position $(0,-2)$. Examining the paths, and assuming straight-line travel, we calculate that in 5 minutes, the Seacar will reach position $(6,-6)$, while the Oceancycle will be at position $(10,13)$.

   (a) We can use parametric equations to describe the paths of each vehicle. For instance, the x coordinate of the Seacar is modeled by $x = t+1$. Can you see why? Use the remaining information to determine the y coordinate of the Seacar, as well as the x and y coordinates of the Oceancycle. (Hint: x and y are each dependent variables, and t is the independent variable. Use this and the given coordinates to find the necessary linear equations.)

Seacar: (1,4) @ t=0, (6,-6) @ t=5

at t=0: x=1 → slope: 5/5 = 1. ⇒ x−1=1(t−0) ⇒ x=t+1.
at t=5: x=6

at t=0: y=4 } slope −10/5 = −2 ⇒ y−4=−2(t−0) ⇒ y=−2t+4.
at t=5: y=−6

Oceancycle:
t=0, x=0 → slope = 2   x = 2t
t=5, x=10
t=0, y=−2 → slope = 3   y+2 = 3(t−0) ⇒ y = 3t−2
t=5, y=13

   (b) Using the parametric equations you found above, do the two vehicles' paths intersect? (If so, when and where?) Do they collide? S[t_] = {t+1, −2t+4}, O[t_] = {2t, 3t−2}

Solve[S[t] == O[r], {t,r}]
t = 9/7, r = 8/7

Do intersect, do not collide.
Intersect at (2.286, 1.429)

2. You and your brother are spending a week at the lake. (Yes, pretend it's summer!) You've each gathered a couple friends and are on opposite sides of the lake, ready to try out your new water balloon launchers. You've done all your calculations, and each of your teams is perfectly positioned to soak the other once the balloons land. The path of your water balloon is given by the parametric equations

M: $$x = 200\cos(30°)t + 40,\ y = -4.9t^2 + 200\sin(30°)t + 50,$$

and the path of your brother's water balloon is given by

B: $$x = 200\cos(165°)t + 4040,\ y = -4.9t^2 + 200\sin(165°)t + 450.$$

   (a) Plot the paths of the two water balloons in Mathematica, and then animate the motion of the two balloons together. (Note, to let Mathematica know you want to use degrees instead of radians, you can type `Sin[30Degree]`, or find the degree symbol in the palette.) Given your animation, do you and your brother both get hit, or do the balloons collide in mid-air?

Doesn't seem they will collide (though they will intersect sometime between t=0 + 25)

   (b) Verify your above conclusions mathematically.

2 solutions, but only one w/ positive time value:

M(s), B(t)   s = 13.0786,   t = 8.979

Location: ~(2305, 520)

3. Romeo and Juliet had a terrible argument and both decided to walk off their frustration. Romeo walked along a path described by the parametric equation

$$R(t) = \langle 0.5t^2 \sin(2t), (1.5 - e^{-t^2}) \cos(t) \rangle,$$

where time is measured in hours. Juliet walked along a path described by the parametric equation

$$J(t) = \langle -t^2 \sin(t) \cos(t), (1.5 - e^{-t^2}) \cos(t) \rangle,$$

where time is measured in hours.

(a) Plot Romeo and Juliet's paths on the same set of axes.

(b) When do Romeo and Juliet make up?

$t = 1.5708$, collision.

Important Note:
The Solve command fails here, so we must use FindRoot.