

MA104 - Differential Calculus

Lesson 43: Functions of Several Variables

1. Let $f(x, y) = \sin(xy) + 2x$.
 - (a) Define the function in Mathematica.
 - (b) Find $f(1, 0)$.
 - (c) Use the `Plot3D` function to generate a graph of the function for $0 \leq x \leq \pi$ and $0 \leq y \leq \pi$. Don't forget to label the axes!
 - (d) Now add the option `ViewPoint -> {1, 3, 5}` to the `Plot3D` command. What happened to the graph?

2. Let $g(x, y) = \frac{-3y}{x^2 + y^2 + 1}$.
 - (a) Define the function in Mathematica.
 - (b) Use the `ContourPlot` function to generate a graph of the function for $-5 \leq x \leq 5$ and $-5 \leq y \leq 5$.
 - (c) Estimate the function value on each of the level curves. Based on the values in the contour plot, describe the behavior of the function.

 - (d) Use the `Plot3D` function to generate a graph of the function. Does it look like what you expected?

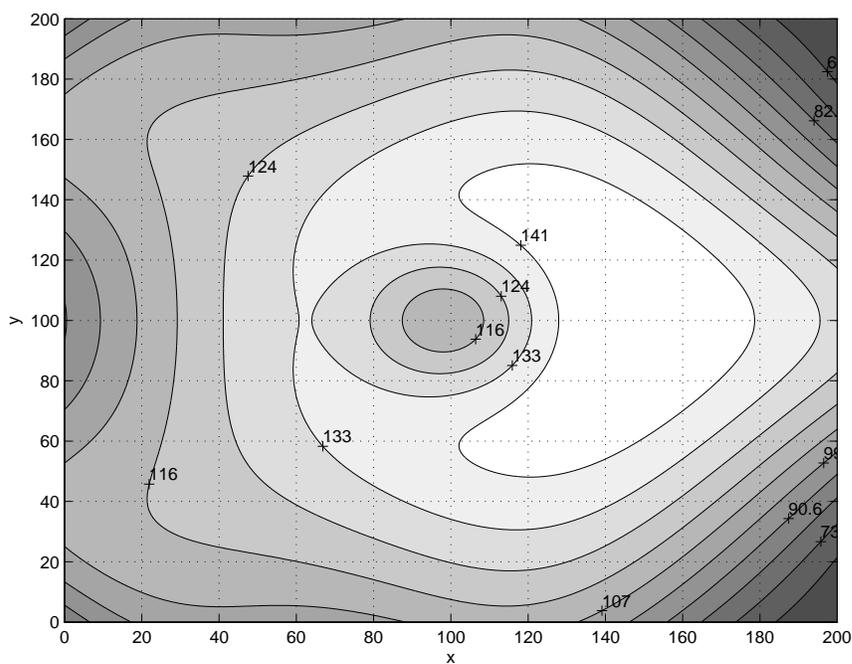
3. Remind yourself: what are the three rules you must remember when finding the domain of a function?

4. Find the domain of the given function, then sketch the domain.
 - (a) $f(x, y) = 1 - \frac{\cos(x^2 - 2)}{x^2 + y^2}$

(b) $f(x, y) = \sqrt{3 - x^2 - y^2}$

(c) $f(x, y) = \frac{1}{1 - (x^2 + y^2)} + \sqrt{1 - (x^2 + y^2)}$

5. Below is a contour plot of the function $g(x, y)$. Use it to estimate $g(150, 60)$, $g(40, 20)$ and $g(40, 160)$. If this function represented elevation of land, how would you describe the topography?



6. Go to <http://www.weather.gov/om/windchill/index.shtml>. Let $f(v, T)$ be the wind chill index ($^{\circ}F$) and $FB(v, T)$ be the time it takes to get frostbite in minutes when the wind speed is v mph and the temperature is T ($^{\circ}F$).
- (a) What is $f(35, -10)$? In the physical context of this problem, what does this number represent?
- (b) Keeping in mind the physical context of this problem, describe in words the meaning of the question “For what value of v is $f(v, -20) = -51$ ”? Then answer the question.
- (c) What is the meaning of $FB(40, T)$? Describe the behavior of this function.
- (d) The actual “wind chill function” is given on the website. Plot f over the region given in the chart online using the Mathematica command “Plot3D”, then plot the same function using “ContourPlot”.