MA386 - Introduction to Numerical Analysis
Homework Assignment 7, 50 points
Due by COB Nov 13

1. Solve the following system using three digit chopping arithmetic using Gaussian elimination with (a) no pivoting and then with (b) scaled partial pivoting. Remeber to keep your work neat and double-check as you go (it would be a good idea to work this with a buddy and compare answers at each step).

$$\begin{bmatrix} 500 & 10000 & 9 \\ 4 & 10 & -1 \\ 10 & 9 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 20 \\ -3 \\ 40 \end{bmatrix}.$$

   The actual solution to 4 significant digits is $\mathbf{x} = [0.4309, -0.02359, 4.488]^T$. Which of your answers from (a) or (b) is closer to the actual solution? Does this surprise you? Why or why not?

2. Section 6.5, 2 b (by hand).

3. Section 6.5, 5 a (by hand). Note: If you're trying to check your work, keep in mind that the `MATLAB` command `[L,U,P] = lu(A)` uses partial pivoting.

4. Section 6.5, 8 a (by hand). Use your answer from the previous problem to solve the system.

BONUS The following $n \times n$ tridiagonal matrix comes up when solving differential equations numerically.

$$A = \begin{bmatrix} 2 & -1 & 0 & \cdots & \cdots & 0 \\ -1 & 2 & -1 & 0 & \cdots & 0 \\ 0 & -1 & 2 & -1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & -1 & 2 & -1 \\ 0 & \cdots & \cdots & 0 & -1 & 2 \end{bmatrix}$$

Compute the LU decomposition for this matrix for $n = 3, 4, 5$. Make a conjecture on the form of the LU decomposition of these matrices for general n, then prove it. For you MA383 fans out there, this could be good practice of induction.